## Unit: Quadratic Functions

Learning increases when you have a goal to work towards. Use this checklist as guide to track how well you are grasping the material. In the center column, rate your understand of the topic from 1-5 with 1 being the lowest and 5 being the highest. Be sure to write down any questions you have about the topic in the last column so that you know what you have yet to learn. You can rate yourself again before the test to see what you need to focus on.

| Outcomes | Understanding (1-5) | Questions? |
| :--- | :--- | :--- |
| 11P.R.3. Analyze quadratic |  |  |
| functions of the form |  |  |
| $y=a(x-p)^{2}+q$ and determine |  |  |
| the: |  |  |
| - vertex |  |  |
| - domain and range |  |  |
| - direction of opening |  |  |
| - axis of symmetry |  |  |
| - $x$ - and $y$-intercepts. |  |  |
| 11P.R.4. Analyze quadratic |  |  |
| functions of the form |  |  |
| $y=a x^{2}+b x+c$ to identify |  |  |
| characteristics of the |  |  |
| corresponding graph, including: |  |  |
| - vertex |  |  |
| - domain and range |  |  |
| - direction of opening |  |  |
| - axis of symmetry |  |  |
| - $x$ - and $y$-intercepts |  |  |
| 11P.R.5. Solve problems that |  |  |
| involve quadratic equations. |  |  |
| 11P.R.6. Solve, algebraically and |  |  |
| graphically, problems that <br> involve systems of linear- <br> quadratic and <br> quadratic equations in two <br> variables. |  |  |

## Lesson 4.1: Properties of a Quadratic Function

Review of Terms:

A relation is $\qquad$ ( $\qquad$ , __ ).

It can be expressed as:
a)
b)
c) $\qquad$ ex.
d) $\qquad$
e) $\qquad$
ex.

|  |
| :--- |
|  |

星
ex.

2) $A$ $\qquad$ is a relation where only one unique $\qquad$ value (__ ) exists for each $\qquad$ value (__). ). (One input value ( $x$ ) results in only one unique input value).
examples of functions:
examples of relations that are NOT functions:
3) The $\qquad$ test is a visual tool used for determining if a graphical relation
is a $\qquad$ . If a vertical line touches a relation at $\qquad$ , it is not a function.
ex. 1

4) The $\qquad$ is the set of input values (__) for which a function or relation is defined or valid.
5) The $\qquad$ is the set of output values (__) for which a function or relation is defined or valid

- There are many different ways of indicating domain and range:

Set Notation
a) $\qquad$ ( $x$ is part of the set of real numbers)
( $x$ is greater than or equal to 6 )
$\{x \mid x \geq 6\}$
$\{x \mid x \in R, x \geq 6\}$
$x \geq 6$
c) $\qquad$ (greater than -2 or less than 1) (between - 2 and 1)

Rules for what brackets to use:

- When simply listing numbers, use set brackets.

Ex: State the domain and range for the following relation: $\quad\{(1,2),(-2,3),(3,4)\}$

- Square brackets includes the number.

Ex:

- Round brackets does not include the number.

Ex:

Examples: State the domain and range of the following relation. Also, list the ordered pairs.
1)


Assignment: "What did the baby porcupine say when it backed into a cactus?"

## Lesson 4.1 - Properties of a Quadratic Function

## THINK:

You are given the following set of squares with various side lengths


Create a table of values and graph the perimeter of each square:

| Side length ( x ) | Perimeter ( y ) |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



Create a table of values and graph the area of each square:

| Side length (x) | Area (y) |
| :--- | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



How are these two graphs the same? How are they different?

The above two graphs are the same in that they both deal with squares of different lengths. One graph shows the perimeter of a square as a straight line. The other graph shows the area of a square which we see as increasing $\qquad$ . How do we compare these two graphs mathematically?

- The perimeter graph shows a $\qquad$ function. This means that there are
$\qquad$ in the equation. Ex. $y=3 x+1$
- The area graph shows a $\qquad$ function. This means that there is an
$\qquad$ of $\qquad$ in the equation. Ex. $y=x^{2}, y=3 x^{2}-4$

General Form of a Quadratic Equation: $y=a x^{2}+b x+c$ (2 $2^{\text {nd }}$ degree equation)

- where $a, b$, and ${ }^{c}$ are real numbers and $a \neq 0$
- This is called the $\qquad$ of the equation of a quadratic function examples:


## Graphing:

Graph the following equations by: (i) creating a table of values, (ii) plotting the coordinates, and (iii) joining the points with a smooth curve.

| Ex. 1 | $y=x^{2}$ |
| :--- | :--- |

Table of values

| $x$ | $y$ |
| ---: | ---: |
| 3 |  |
| 2 |  |
| 1 |  |
| 0 |  |
| -1 |  |
| -2 |  |
| -3 |  |

Graph


The shape of the graph of a quadratic function is called a $\qquad$

Ex. $2 \quad y=(x-3)^{2}-7$

Table of values



Characteristics of parabola discovery activity:
Pearson CD

Properties of a Quadratic Function

On the Controls tab, set the values of $a, b$, and $c$ by moving the sliders or typing values into the input boxes.

Select the Table tab to view a table of values for the graph of the function.

For the quadratic function that has been defined, identify:

- the coordinates of the vertex
- the $x$ - and $y$-intercepts
- the equation of the axis of symmetry
 Show axis of symmetry


| Graph Window | -9.00 | 9.00 | -9.00 | 9.00 |
| :---: | :---: | :---: | :---: | :---: |
|  | $x$-Min | $x$-Max | $y$-Min | $y$-Max |

The characteristics of a parabola are as follows:
i. The $\qquad$ of a parabola is its highest or lowest point. The vertex may be a $\qquad$ or a $\qquad$
ii. The $\qquad$ intersects the parabola at the vertex. The parabola is $\qquad$ about this line.
iii. When the coefficient of $x^{2}$ is $\qquad$ the parabola $\qquad$ and its vertex is a minimum point. When the coefficient of $x^{2}$ is $\qquad$ the parabola $\qquad$ and its vertex is a maximum point.


Negative:

iv. The $\qquad$ of a quadratic function is all possible $x$-values. The domain is $x \in \mathbb{R}$
v. The $\qquad$ of a quadratic function is all possible $y$-values. It depends on the position of the vertex.

For the following graphs of quadratic functions, state the following characteristics:

Graph \#1 $\quad y=-2 x^{2}-6 x+20$


Vertex: $\qquad$
Axis of Symmetry: $\qquad$
Min / Max at: $\qquad$
Domain: $\qquad$

Range: $\qquad$
X-intercept(s): $\qquad$
Y-intercept: $\qquad$

## Solution:

i) Substitute each value of $x$ in $y=-2 x^{2}-6 x+20$, then determine the corresponding value of $y$.

| $\boldsymbol{x}$ | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{y}$ |  |  |  |  |  |  |  |  |  |  |

Since each $y$-value is a multiple of 4 , use a scale of 1 square to 4 units on the $y$ axis. (use the graph above)
ii) From the table, the $x$-intercepts are -5 and 2. The $y$-intercept is 20.

- From the graph, the coordinates of the vertex are the coordinates of the maximum point. They appear to be: $\qquad$
- Draw a vertical line through the vertex. The line appears to pass through 1.5 on the x-axis. So, the equation of the axes of symmetry appears to be:
- The domain is all possible $x$-values. The domain is $\qquad$
- The range is all possible $y$-values. The greatest $y$-value is the $y$-coordinate of the vertex. So, the range is: $\qquad$

If the x-intercepts cannot be identified from the table or graph, they can be determined by solving the related quadratic equation; that is, the $x$-intercepts are the values of $x$ when $y=0$.

For Graph \#1, solve $0=-2 x^{2}-6 x+20$


Solve by factoring

The $x$-intercepts of the graph of a quadratic function $y=a x^{2}+b x+c$ are called the $\qquad$ .

The zeros of the function are the $\qquad$ of the related quadratic equation $a x^{2}+b x+c=0$.

- The above example was modified from Pearson's PreCalculus 11 myWORKTEXT

Graph \#2 - Try it on your own: $y=2 x^{2}+4 x-6$


Vertex: $\qquad$

Axis of Symmetry: $\qquad$

Min / Max at: $\qquad$

Domain: $\qquad$

Range: $\qquad$

X-intercept(s): $\qquad$

Y-intercept: $\qquad$

Graphing Quadratic Example 3 p. 255-256

Exercises: p. 257-261 \#1, 3, 4, 5, 7, 8, 9, 11.
Multiple Choice: p. 261 \#1-2

Attachment 10.2
Three-Point Approach for Words and Concepts

| Detinition | Word or Concopt <br> vertex | Diagram |
| :--- | :--- | :--- |
| $\square$ <br> $\square$ |  |  |
|  |  |  |
|  |  |  |


| Detinition | Word or Conoept <br> Axis of symmetry | Diagram |
| :--- | :--- | :--- |
| $\square$ |  |  |
|  |  |  |
|  |  |  |


| Dotinition | Word or Concept <br> intercepts | Diagram |
| :--- | :---: | :---: |
| $\square$ |  |  |
|  |  |  |
|  |  |  |


| Detinition | Word or Concept <br> parabola | Diagram |
| :--- | :--- | :--- |
| $\square$ |  |  |
| $\square$ | Synonym/Example |  |
| $\square$ |  |  |

## 4.2/4.3 - Solving a Quadratic Equation Graphically

For the following graphs of quadratic functions, state the following characteristics:

Graph \#1 $y=x^{2}$


Vertex: $\qquad$
Axis of Symmetry: $\qquad$
Min / Max at: $\qquad$
Domain: $\qquad$

Range: $\qquad$
X-intercept(s): $\qquad$
Y-intercept: $\qquad$
Graph \#2 $y=2 x^{2}+2$


Vertex: $\qquad$
Axis of Symmetry: $\qquad$
Min / Max at: $\qquad$

Domain: $\qquad$
Range: $\qquad$
X-intercept(s): $\qquad$
Y-intercept: $\qquad$
Do: Quadratic Functions on the Graphing Calculator
Exercise 2 and 3

## Quadratic Functions on the Graphing Calculator

## Keys On The Graphing Calculator

$Y=\quad$ This is where you will enter the equation that you want to graph

GRAPH This button will show any graphs that have been highlighted from $Y=$

WINDOW This button allows you to change the scales on your graph

TRACE This button places a cursor on your graph to locate certain points
$\mathbf{X}, \mathbf{T}, \theta, \mathbf{n}$ This button enters your variable (usually $x$ ) into your equation to be graphed

## $\wedge$

This button allows you to enter an exponent. For example $x^{2}$ can be entered as $x^{\wedge} 2$

## Graphing Quadratic Functions

Press $Y=$ and clear any equations by highlighting the equation and then pressing CLEAR Enter the equation $y=x^{2}$. Select WINDOW and set $X \min$ to -9.4 and $X \max$ to 9.4.
Select GRAPH to see the graph of $y=x^{2}$.

Use the TRACE button to do the following:
a) Find the value of $y$ when $x=7$.
b) Find the value of $y$ when $x=-0.2$. $\qquad$
c) Find the value of $y$ when $x=-3.4$. $\qquad$
d) Describe how these values could have been found algebraically.

## GROUP A

Go back to the equation screen by selecting $Y=$. Enter the equation $y=x^{2}+4$. Look at the graph by selecting GRAPH.

How does this graph compare to the graph of $y=x^{2}$ ?

Predict how each of the following graphs will compare to the graph of $y=x^{2}$.
i) $y=x^{2}-5$
ii) $y=x^{2}+2$

Graph each of these separately to check your prediction.

In general, given the graph of $y=x^{2}+k$ :
a) Describe the transformations that take place in comparison to $y=x^{2}$.
b) State the vertex.
c) State the axis of symmetry
d) State the domain.
e) State the range.

Group B
Go back to the equation screen by selecting $Y=$

Enter the equation $y=3 x^{2}$. Look at the graph by selecting GRAPH
How does this graph compare to the graph of $y=x^{2}$ ?

Enter the equation $y=\frac{1}{3} x^{2}$. How does this graph compare to the graph of $y=x^{2}$ ?

Enter the equation $y=-x^{2}$. How does this graph compare to the graph of $y=x^{2}$ ?

Predict how each of the following graphs will compare to the graph of $y=x^{2}$.
i) $y=4 x^{2}$
ii) $y=\frac{1}{5} x^{2}$
iii) $y=-2 x^{2}$

Graph each of these separately to check your prediction.

In general, given the graph of $y=a x^{2}$ :
f) Describe the transformations that take place in comparison to $y=x^{2}$.
g) State the vertex.
h) State the axis of symmetry $\qquad$
j) State the range.

Complete this table for the graph of each function:

| Function | Direction of <br> opening | Vertex | Axis of <br> Symmetry | Congruent to <br> $y=x^{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| $y=x^{2}$ | Up | $(0,0)$ | $x=0$ | yes |
| $y=(x-7)^{2}$ |  |  |  |  |
| $y=(x+8)^{2}$ |  |  |  |  |
| $y=x^{2}+6$ |  |  |  |  |
| $y=x^{2}-7$ |  |  |  |  |
| $y=6 x^{2}$ |  |  |  |  |
| $y=-6 x^{2}$ |  |  |  |  |

## 4.4 - Analyzing Quadratic Functions of the form $y=a(x-p)^{2}+q$

What have we learned about graphing quadratic functions so far?
See animations from p. 277 of the Pearson Workbook:

1) Changing $\qquad$ in $\qquad$

The graph of $y=x^{2}+q$ is the image of the graph of $y=x^{2}$ after a $\qquad$ of $q$ units

- When $q$ is $\qquad$ the graph moves $\qquad$ .

- When $q$ is $\qquad$ the graph moves $\qquad$ .

$$
\text { Graph } y=x^{2}, y=x^{2}+2, \text { and } y=x^{2}-2
$$


2) Changing $\qquad$ in $\qquad$
The graph of $y=(x-p)^{2}$ is the image of the graph of $y=x^{2}$ after a horizontal translation of $p$ units.

- A negative $\qquad$ moves the graph to the $\qquad$
- A positive $\qquad$ moves the graph to the $\qquad$

|  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$$
\text { Graph } y=x^{2}, y=(x-3)^{2}, \text { and } y=(x+4)^{2}
$$

3) Changing $a$ in $y=a x^{2}$

- The graph of $y=a x^{2}$ is the image of the graph of $y=x^{2}$ after a
$\qquad$
- The graph of $y=a x^{2}$ is the image of the graph of $y=x^{2}$ after a
$\qquad$ of factor a when $\qquad$ .


$$
\text { Graph } y=x^{2}, y=2 x^{2} \text {, and } y=0.5 x^{2}
$$

- The graph of $y=a x^{2}$ is the image of the graph of $y=x^{2}$ after $a$ $\qquad$
$\qquad$ of factor $a$ and a reflection in the $x$-axis when $\qquad$
- The graph of $y=a x^{2}$ is the image of the graph of $y=x^{2}$ after $a$ $\qquad$
$\qquad$ of factor $a$ and a reflection in the $x$-axis when $\qquad$ .


These three transformations combined to form the $\qquad$ of the equation of a quadratic function: $\qquad$

Example 1: Graph $y=2(x-2)^{2}+2$


Example 2: Graph $y=-0.5 x^{2}-6$


Example 3: Graph $y=2 / 3(x+2)^{2}$


Exercises 4.4 p. 284
Quadratic Functions Assignment

## Quadratic Functions Assignment

For this assignment you will solve the questions labeled by the first three different letters of your last name.

For each question determine the: a) vertex; b) the equation of the axis of symmetry;
c) direction of opening; d) domain; e) range; f) relative width (compared to $y=x^{2}$ );
g) max or min value; and h) draw a sketch of the graph.

Provide answers and work on the back of this sheet. Each question is worth six marks.

Due: $\qquad$ , at the beginning of class

| A $y=\frac{1}{2}(x+1)^{2}-4$ | H $y=3 x^{2}+6$ | $y=-2(x-3)^{2}+1$ | $y=-\frac{1}{3}(x-1)^{2}+2$ |
| :---: | :---: | :---: | :---: |
| > B $y=2(x-1)^{2}-6$ | $y=3(x-2)^{2}+6$ | $y=-(x-5)^{2}$ | $\begin{aligned} & \mathbf{V} \\ & y=x^{2}-1 \end{aligned}$ |
| $\begin{aligned} & \mathbf{C} \\ & y=-2 x^{2}+1 \end{aligned}$ | $y=-\frac{1}{2} x^{2}+1$ | $\begin{aligned} & \mathbf{Q} \\ & y=-\frac{1}{2}(x-3)^{2}+2 \end{aligned}$ | $\begin{aligned} & \mathbf{W} \\ & y=(x-2)^{2}+1 \end{aligned}$ |
| $\begin{aligned} & \text { D } \\ & y=(x-2)^{2}+1 \end{aligned}$ | $y=-3(x+5)^{2}-1$ | $\begin{aligned} & \mathbf{R} \\ & y=3(x-2)^{2}-3 \end{aligned}$ | $\begin{aligned} & \mathbf{X} \\ & y=2 x^{2}+5 \end{aligned}$ |
| $y=-(x+5)^{2}+2$ | L $y=-\frac{1}{2} x^{2}+3$ | $\begin{aligned} & \mathbf{S} \\ & y=-(x+6)^{2}+1 \end{aligned}$ | $\begin{aligned} & \mathbf{Y} \\ & y=x^{2}-25 \end{aligned}$ |
| $y=-\frac{1}{2}(x-5)^{2}+1$ | $\begin{aligned} & \mathbf{M} \\ & y=-2(x-3)^{2} \end{aligned}$ | $y=2(x-3)^{2}-8$ | $y=-\frac{1}{3}(x+1)^{2}+3$ |
| $\begin{aligned} & \mathbf{G} \\ & y=-(x-3)^{2}-1 \end{aligned}$ | $y=-3(x+4)^{2}+6$ |  |  |

