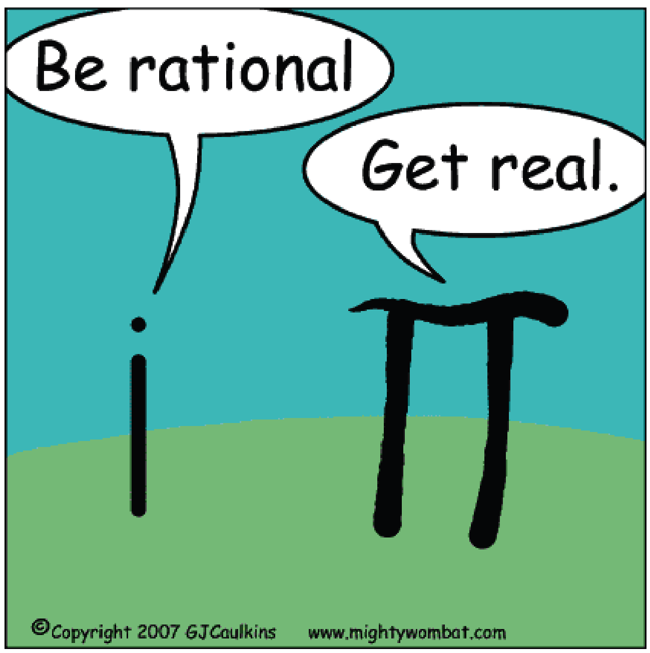
Number Sense
Unit 1



Math 10F
Mrs. Kornelsen
R.D. Parker Collegiate

Lesson One: Rational Numbers

New Definitions:

Rational Number –

Is every number a rational number?

What about the following? Why or why not?

a) 0.3694

b) $0.333333\overline{3}$

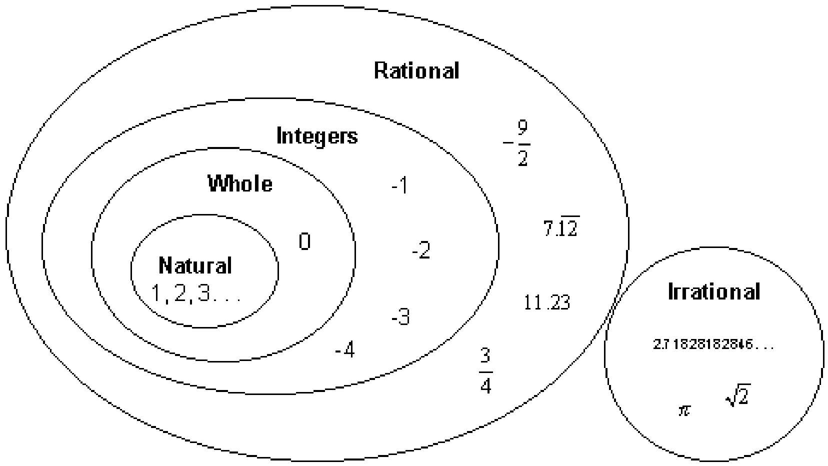
c) $0.31428563523\ldots$

A rational number can be a decimal as long as it ________________________________ or ___________________.

Circle the following rational numbers

$\frac{3}{4}$	12	0.008		$\sqrt{13}$
16.7	$-\sqrt{7}$	π	$\sqrt{64}$	$\frac{1}{3}$

The Real Number System



The real number system consists of natural numbers, whole numbers, integers, rational and irrational numbers.

<u>Natural Numbers (ℕ)</u>
Natural numbers are the ___________________________________ (positive integers)

<u>Whole Numbers (𝕎)</u>
Whole numbers are the counting numbers _________________ (non-negative integers)

<u>Integers (ℤ)</u>
Integers are the natural numbers and their ____________________________________

No ____

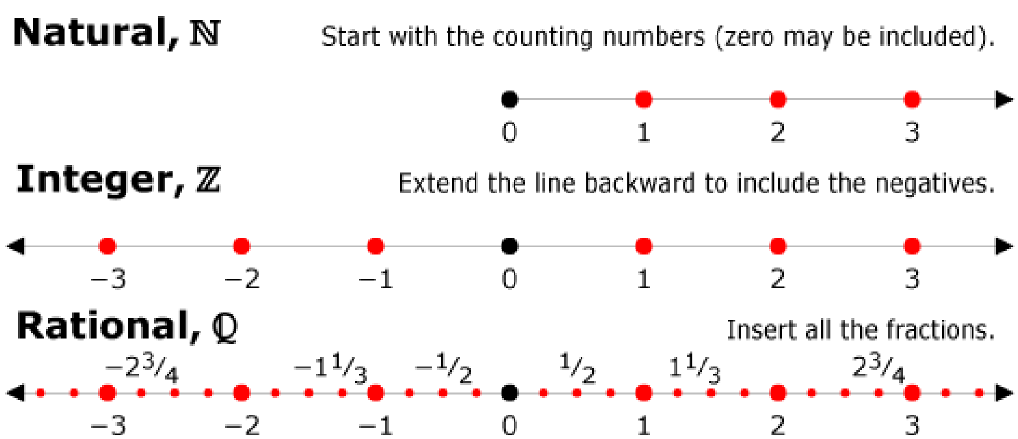
<u>Rational Numbers (ℚ)</u>
A rational number is a number which can be expressed as a ratio/fraction of two integers.

Example:

Irrational Numbers: ($\overline{\mathbb{Q}}$)
The set of numbers that are ____

Examples:

Number Line



Any number that represents an amount of something, such as a ____________________, a ________________________________, or the _________________________________ between two points, will always be a real number.

Identifying and Classifying Numbers

1. Explain the difference between a rational and an irrational number.

Classify the following numbers as rational or irrational.

2. ½   3. 8   4. $\sqrt{6}$ 5. $\sqrt{16}$ 6. π

7. List the set of all natural numbers.

8. List the set of whole numbers less than 4.

9. List the set of integers such that $-3 < x < 5$.

Classify the following numbers as rational, irrational, natural, whole and/or integer. (A number may belong to more than one set)

10. -3 12. $4\frac{2}{3}$ 13. $\sqrt{3}$ 14. 0

15. Using the following set of numbers:
$A = \left\{\sqrt{3.6},\ 0.36,\ -\frac{3}{6},\ 0.3\bar{6},\ 0,\ 3^6,\ -3,\ \sqrt{36},\ 3.63363336\ldots\right\}$, place each element in the appropriate subset. (Numbers may belong to more than one subset)

rational numbers________________________   irrational numbers______________________

natural numbers________________________   whole numbers________________________

integers________________________

True or False?

16. All whole numbers are rational numbers.

17. All integers are irrational numbers.

18. All natural numbers are integers.

The Rational Number System

Classify these numbers as rational or irrational and give your reason.

1. a. 7329

   b. $\sqrt{4}$

2. a. 0.95832758941…

   b. 0.5287593593593

Give an example of a number that would satisfy these rules.

3. a number that is: real, rational, whole, an integer, and natural
4. a number that is: real and irrational
5. a number that is: real, rational, an integer

Classify each number as: real, rational, irrational, whole, natural, and integer. Give your reason.

6. a. 3/4

   b. -12/4

7. a. 0.345 345 345

   b. -0. 6473490424

8. Give examples of rational numbers that fit between the following sets of numbers.

a. -0.56 and -0.65

b. -5.76 and -5.77

c. 3.64 and 3.46

9. Which two numbers are irrational? How do you know?

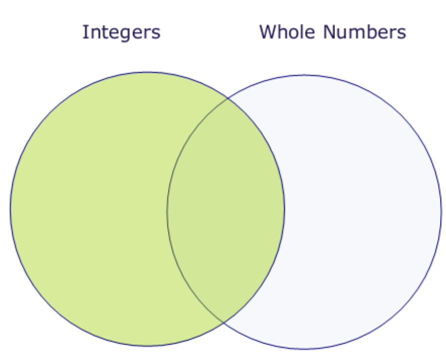
a. $8-\sqrt{56}$

b. $8-\sqrt{25}$

c. $2-\sqrt{73}$

10. Place the following numbers in the Venn Diagram. Place the following numbers in the Venn Diagram. Note that some numbers may not fit in the diagram.

-0.462	-56	735	0.326	8321	0



Fractions

Three types:

1) _____________________: the top number (numerator) is bigger than the bottom number (denominator).

   Ex:

2) _____________________: the top number (numerator) is smaller than the bottom number (denominator)

3) Ex: _________________

4) ____________________________: A whole number and a fraction.

   Ex: ________________

Mixed → Improper

Take the whole number times the denominator and add the numerator. Put that number over the original denominator.

Example:

$2\frac{2}{3}$

$3\frac{2}{11}$

Improper → Mixed example

1) How many times does the denominator go into the numerator? That's your whole number

2) How many are left over? That's your numerator

   - The denominator stays the same

Adding and Subtracting Fractions:

1) Put each fraction over the same _________________________

2) Add or subtract the __________________. Leave the denominator the same.

Ex: $\frac{3}{4}+\frac{7}{8}$

$\frac{11}{12}-\frac{3}{4}$

$\frac{3}{5}+\frac{1}{3}$

$5\frac{1}{5}+3\frac{1}{8}$

Multiplying Fractions:

1) Check to see if you can reduce. Include cross-reducing.
2) Multiply the top x top and the bottom x bottom.
3) Reduce

Ex.

i) $\frac{20}{3}\times\frac{9}{2}=$

ii) $\frac{1}{3}\times\frac{2}{5}=$

iii) $\frac{5}{4}\times\frac{24}{2}=$

iv) 3 x $\frac{3}{10}=$

Do Fractions Handouts
Prepare for Fractions Quiz

Dividing Fractions:

- Flip the second fraction (find the reciprocal)
- Multiply using the steps above:

Ex:

i) $\frac{20}{3}\div\frac{9}{2}$

ii) $\frac{1}{3}\div\frac{2}{9}$

iii) $4\frac{1}{3}\div 1\frac{2}{9}$

iv) $3\frac{1}{4}\div 2$

Lesson Two: Rational Numbers In Between Numbers

You can always find a number that fits in-between other numbers

Example 1:

Write a number in the blank space in each decimal so that the top decimal is greater than the bottom decimal.

a) $0.239\_\_$
$0.239\_\_$

b) $11.2\_\_5$
$11.2\_\_5$

Example 2:

Put a number in the blank space in each fraction, so that the top fraction is greater than the bottom fraction

a) $\frac{}{8}$

$\frac{}{8}$

b) $\frac{9}{}$

$\frac{8}{}$

Example 3:

Fill in the missing digits so that the value in the middle decimal is between that of the top and bottom decimal

a) 0.3174

$0.3\_\_\ \_\_\ \_\_$

0.2968

b) 0.3174

$0.\_\_\ \_\_\ \_\_$

0.3000

Example 4:

Identify a fraction between -0.6 and -0.7

Example 5:

Identify a fraction between the following:

$\frac{2}{3}$ and $\frac{5}{6}$

Task

1. a. In each of the following number pairs, put numbers in the blank spaces which will make the top number **greater than** the bottom number:

3.2__0	3.__1__	__.__36
3.2__9	3.2__ __	__.__35

b. In each of the following number pairs, put numbers in the blank spaces which will make the top number l**ess than** the bottom number:

3.2__0	3.__1__	__.__36
3.2__9	3.2__ __	__.__35

2. a. In each of the following fraction pairs, fill in the blank spaces so that the top fraction is **greater than** the bottom fraction:

$\frac{\_\_}{7}$	$\frac{\_\_}{30}$
$\frac{4}{5}$	$\frac{5}{8}$

b. In each fraction pair, fill in the blank spaces so that the top fraction is l**ess than** the bottom fraction:

$\frac{\_\_}{7}$	$\frac{\_\_}{30}$
$\frac{4}{5}$	$\frac{5}{8}$

3. Given the following sets of decimals, fill in the spaces so that, in each group, the value of the middle decimal is **between** that of the top and the bottom decimals:

0.3174	0.31__4
0.29__ __	0.__ __ __ __
0.2968	0.2 __ 68

4. Write a number in each box to make the following inequalities true:

a. $\frac{9}{13} < \boxed{} < \frac{10}{13}$

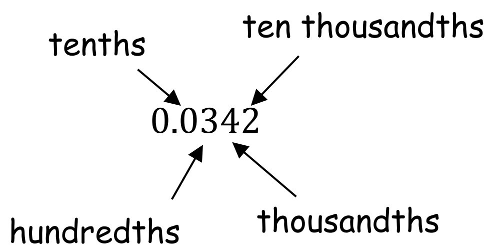
b. $\frac{13}{10} < \boxed{} < \frac{13}{9}$

c. $\frac{9}{13} < \boxed{} < \frac{90001}{130000}$

Lesson Three: Comparing Rational Numbers

How to compare decimals

Know your place values



Example 1:

Order from least to greatest:

0.0342 0.031 0.04 0.03 0.029999

In the hundredths place value there is a 2, 3, 3, 3 and 4. Therefore 0.029999 is the smallest because 2 is smaller than 3. And 0.04 is the biggest because 4 is bigger than 3.

From the three decimals left, there is a 0, 1 and 4 in the thousandths place value. Therefore 0.0342 is the second largest because 4 is bigger than 1 and 0.

How to compare fractions
Turn them into a decimal

$\frac{5}{6} = 0.8\overline{3}$

On your calculator press $\boxed{5}$ then press $\boxed{\div}$ then press $\boxed{6}$ and $\boxed{=}$

To turn mixed numbers in to decimals you divide the fraction and add the whole number.

$-2\frac{2}{5} = -1.6$

On your calculator press $\boxed{2}$ then press $\boxed{\div}$ then press $\boxed{5}$ and $\boxed{=}$

Then press $\boxed{+}$ then $\boxed{-2}$ and $\boxed{=}$

Example 2:
Compare and order from least to greatest:

$-1.2 \qquad \frac{4}{7} \qquad \frac{2}{5} \qquad \frac{9}{16} \qquad \frac{4}{5} \qquad \frac{7}{8} \qquad -0.\overline{5} \qquad -\frac{7}{8}$

Example 3:
Which fraction is greater? $-\frac{3}{4}$ or $-\frac{2}{3}$

Do Comparing Rationals Assignment
Do Comparing Rationals Exit Slip

Comparing and Ordering Rational Numbers **Name**___________________

Fill in each blank with < , > , or = to make each sentence true. Write the decimal form beneath each fraction to check your answer.

Example: $\frac{1}{2} > \frac{1}{3}$

0.5 > 0.333

On your calculator, press $\boxed{1}$ press the $\boxed{\div}$ button, then press $\boxed{2}$ and $\boxed{=}$ to get the decimal form of $\frac{1}{2}$

1. $\frac{2}{3}$ ___ $\frac{5}{8}$

2. $\frac{3}{4}$ ___ $\frac{5}{7}$

3. $\frac{4}{15}$ ___ $\frac{5}{19}$

4. $\frac{3}{14}$ ___ $\frac{15}{70}$

5. $\frac{14}{5}$ ___ $\frac{30}{13}$

6. $\frac{3}{5}$ ___ $\frac{7}{8}$

7. $\frac{7}{10}$ ___ $\frac{15}{19}$

8. $\frac{5}{12}$ ___ $\frac{3}{16}$

9. $\frac{5}{2}$ ___ $\frac{10}{4}$

10. $\frac{4}{13}$ ___ $\frac{3}{9}$

11. $\frac{7}{9}$ ___ $\frac{5}{7}$

12. $\frac{9}{7}$ ___ $\frac{7}{4}$

Write the fractions in order from least to greatest. Write the decimal notation beneath each fraction as you did in problems 1 - 12.

13. $\frac{3}{8}$ $\frac{1}{4}$ $\frac{7}{8}$

14. $\frac{16}{19}$ $\frac{17}{20}$ $\frac{18}{21}$

15. $\frac{3}{5}$ $\frac{18}{29}$ $\frac{24}{39}$

Lesson Four: Operations with Decimals

Example 1:

Estimate and calculate

a) $2.65 + (-3.81)$

b) $-5.96 - (-6.83)$

Example 2:

Estimate and calculate

a) $0.45 \times (-1.2)$

b) $-2.3 \div (-0.25)$

Example 3:

On Saturday, the temperature at the Blood Reserve near Stand Off, Alberta decreased by 1.2° Celsius per hour for 3.5 hours. It then decreased by 0.9° C/h for 1.5 hours.

a) What was the total decrease in temperature?

b) What was the average rate of decrease in temperature?

Lesson Five: Operations with Fractions

Example 1:

Estimate and calculate

c) $\frac{2}{5} - \left(-\frac{1}{10}\right)$

d) $3\frac{2}{3} + \left(-1\frac{3}{4}\right)$

Example 2:

Estimate and calculate

c) $\frac{3}{4} \times \left(-\frac{2}{3}\right)$

d) $-1\frac{1}{2} \div \left(-2\frac{3}{4}\right)$

<u>Example 3:</u>

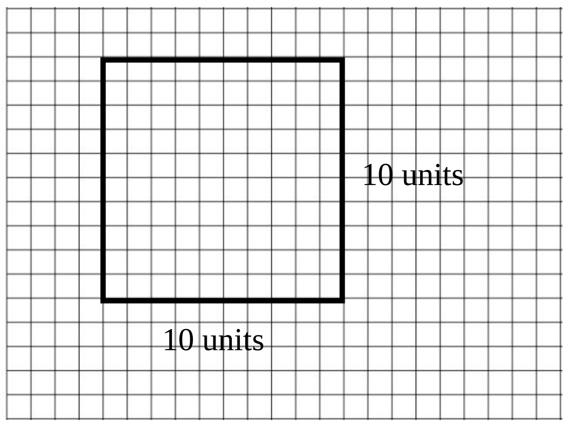
At the start of a week, Maka had $30 of her monthly allowance left. That week, she spent $\frac{1}{5}$ of the money on bus fares, another $\frac{1}{2}$ shopping, and $\frac{1}{4}$ on snacks. How much did she have left at the end of the week?

Lesson Six: Squares and Square Roots

A square is a four-sided figure that has the same length and width. It is a perfect square.

Ex. The length and width of this square are 10 units.

What is the area of this square?



Since $10 \times 10 = 100$ units^2, we can say that $\sqrt{100} = 10$ and is a perfect square.

Think About It!!!

What does $(-5)^2$ equal?

Recall your multiplication of negative integers

The square root of a number has two answers: a positive and a negative
Which of these are perfect squares? Evaluate the perfect squares.

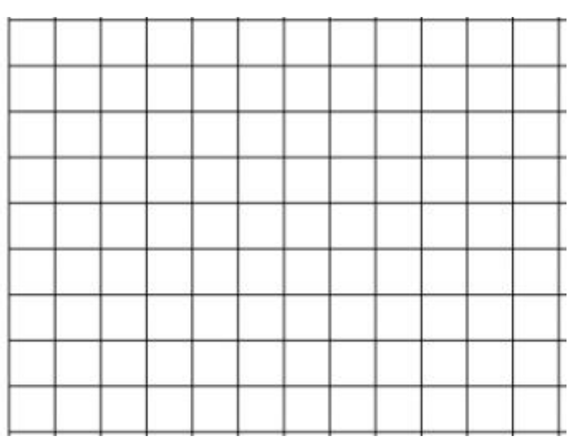
1. $\sqrt{25}$ 2. $\sqrt{142}$ 3. $\sqrt{81}$ 4. $\sqrt{40}$ 5. $\sqrt{16}$

6. $\sqrt{49}$ 7. $\sqrt{10}$ 8. $\sqrt{95}$ 9. $\sqrt{36}$ 10. $\sqrt{4}$

List the perfect squares from 1 to 100.
Starting with $\sqrt{1} = 1$ and ending with $\sqrt{100} = 10$.

<u>Example 1:</u>

Draw a diagram that represents $\sqrt{25}$



Example 2:

Determine whether each of the following numbers is a perfect square.

a) $\frac{25}{49}$

b) 0.4

Example 3:

Evaluate $\sqrt{1.44}$

Lesson Seven: Estimating Square Roots

To estimate the square root of a number, find the perfect squares on each side of the number.

Example 1:

Estimate $\sqrt{89}$

Think: What two perfect squares are on either side of 89.

Try It!!!

Estimate each square root.

1. $\sqrt{20}$
2. $\sqrt{3}$
3. $\sqrt{40}$
4. $\sqrt{110}$
5. $\sqrt{15}$
6. $\sqrt{99}$
7. $\sqrt{53}$
8. $\sqrt{61}$
9. $\sqrt{17}$

Example 2:

A square trampoline has a side length of 2.6 m. Estimate and calculate the area of the trampolinee.

Example 3:

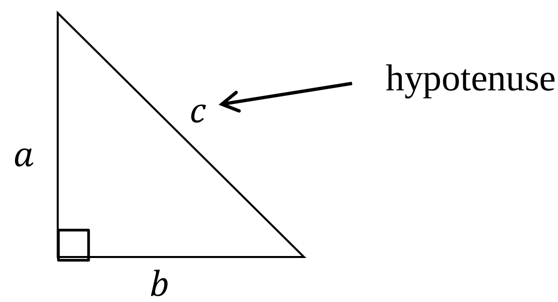
Estimate and then calculate $\sqrt{0.73}$

Lesson Eight: Pythagorean Theorem

Recall:

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

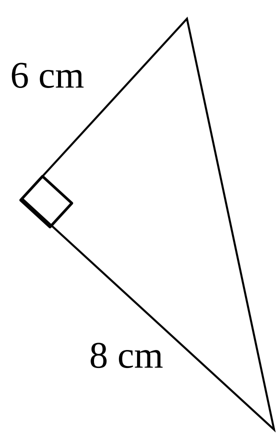


It doesn’t matter what side is a or b.

But c must be the longest side, and the longest side is the side across from the right angle called the hypotenuse.

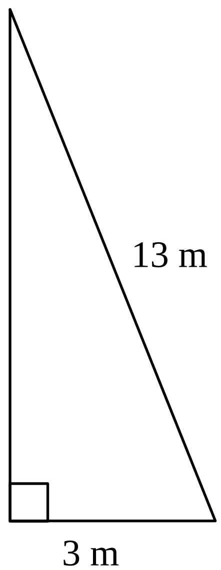
Example 1:

Solve for the missing side.



Example 2:

Solve for the unknown side. Round your answer to the nearest hundredth.

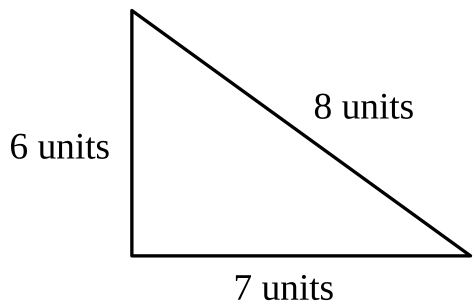


Example 3:

A 3m ladder is leaning against a wall. The base of the ladder is 0.5m from the wall. How far up the wall does the top of the ladder reach? Round your answer to the nearest hundredth of a metre. Provide a diagram.

Example 4:

Determine whether the given triangle is a right triangle.



Irrational Numbers

Irrational Number – a number thata is not rational
It cannot be expressed as a fraction
It cannot be a terminating or repeating decimal

The most common irrational number is $\pi = 3.14159265358979323846264338 3$ ...

Square roots that are not perfect squares are also irrational numbers, such as $\sqrt{2}$.

Calculate $\sqrt{2}$

How was $\sqrt{2}$ discovered?

$a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$2 = c^2$

$\sqrt{2} = c$