Powers and Exponents

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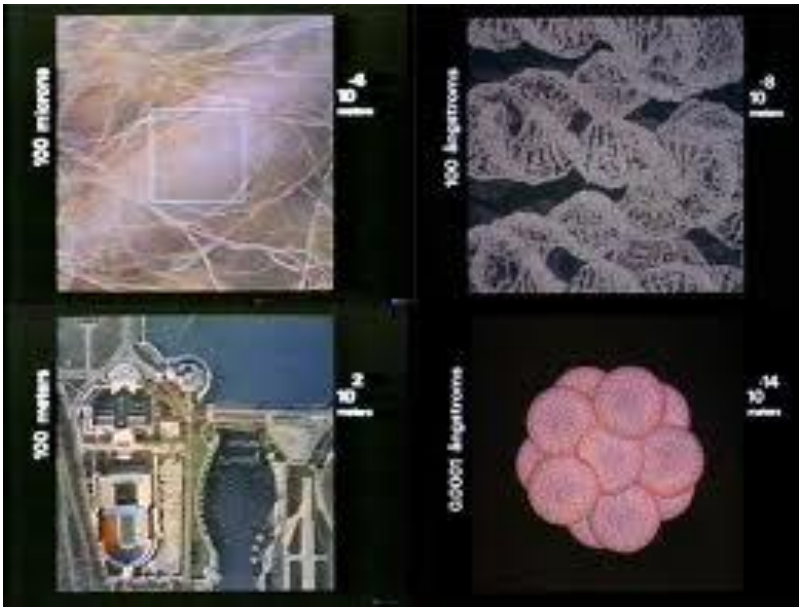


Lesson One: Understanding Powers and Exponents

We write $5 + 5 + 5 + 5$ as 5×4

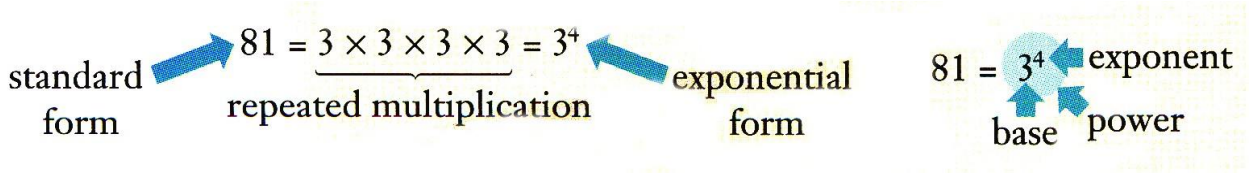
How do we write $8 + 8 + 8 + 8 + 8$?

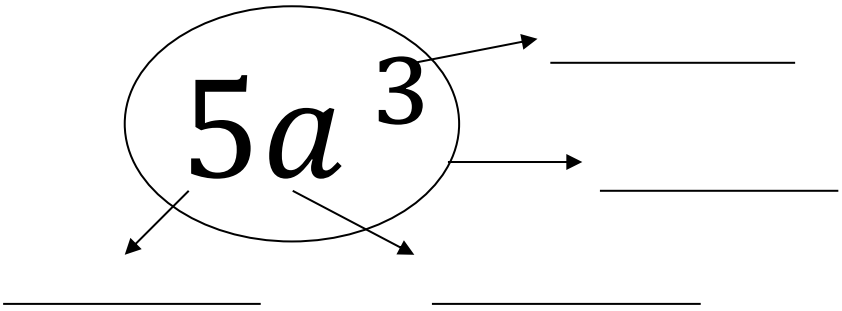
How do you think we write $7 \times 7 \times 7$?

This is read as "seven to the power of three"

The number "7" tells us which factor is repeated

The little number "3" tells us how many times "7" is repeated





Base:

Exponent:

Power:

Coefficient:

For the following name the base, exponent and coefficient:

1) $3x^4$

2) $12y^5$

3) $-2c^8$

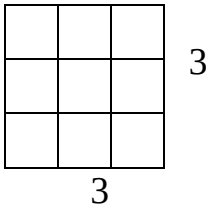
Complete the following table:

Factors	Exponential Notation	Written	Value
		Three to the power of three	
	4^3		
6 x 6 x 6 x 6			
	120^4		
		Five squared	
3 x 3			

Squared

When a base is raised to the second power we say that it is "squared."

$$3^2 = 3 \times 3 = 9$$

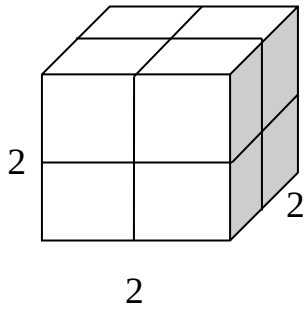


The length and width of a square are equal, in this case they are both 3 units.

Cubed

When a base is raised to the third power we say that it is "cubed."

$$2^3 = 2\ x\ 2\ x\ 2 = 8$$



The length, width and height of a cube are all equal, in this case they are all 2 units.

Special Cases:

- Any base to the power of 1 is the base (ex: $5^1 = 5$)
- Anything to the power of 0 is 1(ex:$9^0 = 1$, $1983^0 = 1$, $(-9)^0 = 1$, $-9^0 = -1$)
- An negative base with an even exponent is positive. A negative base with an odd exponent is negative (ex: $(-5)^3 = -125$, $(-5)^4 = 625$)

Example 1:

Write $2 \times 2 \times 2 \times 2 \times 2$ in exponential form and evaluate it.

Example 2:

Evaluate each power

a) 4^2 b) 2^3 c) 3^6

d) $(4.7)^4$ e) $\left(\frac{3}{4}\right)^2$

Example 3:

Exponents are also used with variables. Write the following as repeated multiplication.

a) $3y^4$

b) $\boldsymbol{xy^2}$

Example 4:

If $\boldsymbol{x = 2}$ and $\boldsymbol{y = -3}$, evaluate $\boldsymbol{5x^4 + 6xy}$.

Lesson Two: Positive and Negative Exponents with Parentheses

Review:

Evaluate the following:

2^3 $(5.2)^3$ $\left(\frac{2}{3}\right)^4$

Negative Bases

Let's look at the following power

$$(-2)^4$$

The exponent is _____.

The base is _____.

The exponent applies to what is _____ beside it. In this case a _____; meaning, everything _____ the bracket must be to the power of 4, including the _____.

Write $(-2)^4$ using repeated multiplication

Evaluate $(-2)^4$

Now let's take a look at the following power:

$$-2^4$$

The exponent is _____.

The base is _____.

The exponent applies to what is _____ beside it. In this case the 2; meaning, _____ the 2 must be to the power of 4, _____ the negative.

Example 1:

Write the following using repeated multiplication and then evaluate:

a) $(-2)^4$

b) -2^4

c) $(-4)^3$

d) $-(-5)^6$

2.1 – Bases, Exponents and Powers

A. Circle the base in each question below.

1. 7^2 2. 8^3 3. 10^5 4. $(3/8)^4$ 5. 1.63^2 6. 87^{25}

B. Circle the exponent in each question below.

1. 7^2 2. 8^3 3. 10^5 4. $(3/8)^4$ 5. 1.63^2 6. 87^{25}

C. Fill in the missing factors.

1. $6^2 = 6 \times$ _____
2. _____ $= 2 \times 2 \times 2 \times 2 \times 2$
3. $7^3 = 7 \times$ _____
4. _____ $= 3 \times 3 \times 3$
5. $5^4 =$ _____ $\times 5$
6. $3^0 =$ _____

D. Express each in exponential form.

1. $3 \times 3 \times 3 \times 3$
2. $2 \times 2 \times 2 \times 2 \times 2$
3. $5 \times 5 \times 5 \times 5 \times 5 \times 5$
4. $10 \times 10 \times 10 \times 10 \times 10$
5. $8 \times 8 \times 8$
6. $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$
7. $-6 \times -6 \times -6 \times -6$
8. $3 \times 3 \times 3 \times 3 \times 3 \times 3$
9. $-(7 \times 7 \times 7 \times 7)$
10. $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$

E. Write each in word form.

1. 7^5
2. 3^2
3. 9^3
4. 2^4
5. 8^0

F. Complete the following table.

Exponential Form	Base	Exponent	Word Form	Factored Form	Standard Form
1. 3^4					
2.	5	3			
3.			eight squared		
4.				$5 \times 5 \times 5 \times 5$	
5.					49

G. Find the value (standard form) of each. The first one is already completed.

1. $5^2 = 5 \times 5 = 25$
2. 8^0
3. 7^3
4. 4^4
5. 10^3
6. two to the eighth power
7. six cubed
8. 16^2
9. 1^{100}
10. 100^4
11. Four squared
12. Ten to the fifth power

2.1 – Exponential and Standard Form Worksheet

A. Write each as a power. The first one is already completed.

1. $36 = 6^2$
2. 7
3. 100
4. 49
5. 25
6. 27
7. 81
8. 16
9. 1
10. 32
11. 125
12. 128

B. What is the value of '*b*' (the base) in each?

1. $b^2 = 36$
2. $b^2 = 100$
3. $b^0 = 1$
4. $b^4 = 81$
5. $b^5 = 32$
6. $b^1 = 30$
7. $b^3 = 64$
8. $b^3 = 1$
9. $b^2 = 49$

C. What is the value of '*e*' (the exponent) in each?

1. $25 = 5^e$
2. $81 = 3^e$
3. $1 = 7^e$
4. $36 = 6^e$
5. $512 = 8^e$
6. $256 = 2^e$
7. $1000 = 10^e$
8. $625 = 5^e$
9. $729 = 3^e$

D. Write each of the following in standard form.

1. $(-2)^2$
2. -2^2
3. $(12)^2$
4. $(-12)^2$
5. -12^2
6. $(\frac{2}{3})^3$
7. 7^0
8. $\frac{2^3}{3}$
9. $(0.3)^4$
10. -0.3^4
11. $(5.2)^3$
12. 5.3^3
13. -1^{50}
14. $(-1)^{50}$
15. $(\frac{7}{8})^3$
16. $(0.01)^2$

E. Circle the expression with the larger value. Show work to justify your answer.

1. 5^2 , 2^5

2. 4^3 , 5^2

3. $(\frac{1}{4})^2$, $(\frac{1}{2})^4$

4. 3^4 , 4^2

5. 4^2 , 2^4

6. $(6-4)^2$, 3^3

7. 8^2 , 2^8

8. 10^3 , $10^2 \times 10^1$

F. Complete the following chart.

Exponential Form	Factored Form	Standard Form
1. 5^2		
2.	$9 \times 9 \times 9$	
3.		81
4.	$3 \times 3 \times 3 \times 3$	
5. $(\frac{2}{3})^3$		
6. $(-7)^2$		
7. -7^2		
8.		16
9.	(0.4)(0.4)(0.4)	
10. 8^0		
11. $(-\frac{2}{3})^3$		
12.		$^{16}/_{25}$.
13.	(1.5)(1.5)(1.5)(1.5)	
14. $(1)^2$		
15. $(-6)^4$		

Exponent Laws

The letters a, b, m and n are coefficients.
Coefficients are constant factors (numbers).

Law 1: Product Law

$$(a^m)(a^n) = a^{m+n}$$

Law 2: Quotient Law

$$a^m \div a^n = a^{m-n}$$

$$m > n$$

Law 3: Power of a Power Law

$$(a^m)^n = a^{mn}$$

Law 4: Power of a Product Law

$$(ab)^m = a^m b^m$$

Law 5: Power of a Quotient Law

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$b \neq 0$$

Law 6: Exponent of Zero

$$a^0 = 1$$

$$a \neq 0$$

Law 7: Negative Exponents

$$a^{-n} = \frac{1}{a^n}$$

$$a \neq 0$$

Lesson Three: Product and Quotient Law of Powers

Product of Powers

Consider the product:

$$x^2 \cdot x^4$$

Write using repeated multiplication

In general we have the product of powers property:

When multiplying powers having the same base, simply add the exponents.

Example 1:

Write each product as a single power. Then, evaluate the power (if possible).

a) $2^3 \times 2^2$

b) $(-3)^2 \times (-3)^5$

c) $x^3 \cdot x^5$

d) $y^2 \cdot y^1 \cdot y^7$

e) $a^3 b^2 a^3$

Quotient of Powers

Consider the quotient"

$$\frac{x^5}{x^3}$$

Write using repeated multiplication

In general we have the quotient of powers property:

When dividing powers having the same base, simply subtract the exponents.

Example 2:

Write each quotient as a single power. Then, evaluate the power (if possible).

a) $2^6 \div 2^2$

b) $(-5)^9 \div (-5)^6$

f) $x^3 \div x^5$

g) $\frac{y^2 y^1}{y^7}$

h) $\frac{a^3 b^2}{a^3}$

Lesson Four: Product and Quotient Law of Powers

Power of a Power

Consider the power

$$(x^2)^3$$

In general, we have the power of a power property:

Example 1:

Write each as a single power. Then, evaluate the power (if possible).

a) $(2^3)^2$

b) $(3^2)^4$

c) $(a^3)^5$

Power of a Product

Consider the power

$$(2x)^2$$

The base of the power $(2x)^2$ is _____.

The entire base is affected by the exponent _____. So the factor 2 must be _____, and the factor x must also be _____.

In general, we have the power of a product property:

Example 2:

Write each as the product of two powers. Then, evaluate.

a) $[2 \times (-3)]^4$

b) $[(-1) \times (-2)]^3$

c) $(3a)^3$

d) $(4x^2)^4$

e) $(x^2y^3)^5$

Power of a Quotient

Consider the quotient

$$\left(\frac{x}{y}\right)^2$$

In general, we have the power of a quotient property:

Example 3:

Write each expression as the quotient of two powers. Then, evaluate.

a) $\left(\frac{3}{4}\right)^3$

b) $\left(\frac{-2}{5}\right)^2$

c) $\left(\frac{2x}{z}\right)^4$

d) $\left(\frac{3z}{6y}\right)^2$

e) $\left(\frac{2xy}{4zyx^3}\right)^2$

Lesson Five: Exponent of Zero

Introduction

Evaluate 3^0 using a table

Power	Value
3^4	
3^3	
3^2	
3^1	
3^0	

Determine the pattern in the values

Consider the quotient $\left(\frac{x^4}{x^4}\right)$

In general, we have the zero exponent property:

$$a^0 = 1 \quad , \quad a \neq 0$$

Example 1:

Evaluate each expression

a) $(-5)^0$

b) -5^0

c) $-(5)^0$

d) 5^0

e) $(5xy^0z)^2$

f) $(5xy^2z)^0$

Lesson Six: Order of Operations

BEDMAS

BEDMAS is the order in which you perform operations like addition, multiplication and division.

B -
E -
D -
M -
A -
S -

Example 1:

Evaluate:

a) $3(2)^4$

b) $-3(-5)^2$

c) -4^4

Example 2:

Evaluate:

a) $4^2 - 8 \div 2 + (-3^2)$

b) $-2(-15 - 4^2) + 4(2 + 3)^3$

Lesson Seven – Solving Problems with Exponents

Introduction:

A population of 10 000 mountain pine beetles doubles each year.

1) Create a table to show the growth of the population of pine beetles over 3 years.

Years	Pine Beetles

2) Express the population each year as a product of 10 000 and a power of 2.

3) What patterns do you notice?

4) How could you determine the number of beetles in ten years without extending the table?

5) How would your table be different if the beetles tripled in number each year?

Example 1:

Write an exponential expression to solve each problem

d) What is the surface area of a cube with an edge length of 4cm?

e) A circle is inscribed in a square with a side length of 20cm. What is the area of the shaded region?

Example 2:

A dish holds 100 bacteria. It is known that the bacteria double in number every hour. How many bacteria will be present after each number of hours?

a) 1
b) 5
c) n