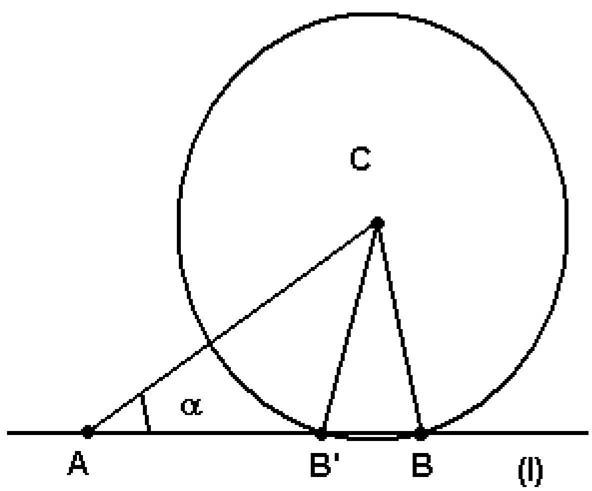
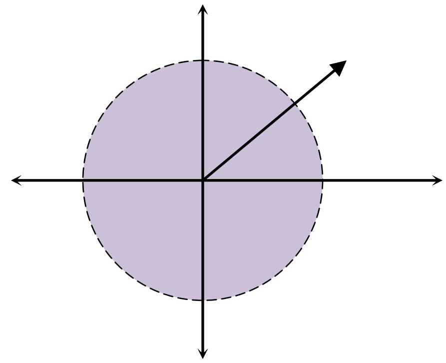
Trigonometry



Lesson 1: Trigonometry – Angles and Quadrants



An angle of rotation can be determined by rotating a ray about its endpoint or ________________. The starting position of the ray is the _______________ side of the angle. The position after rotation is the ___________________ side. If the rotation is __________________-clockwise, the direction is _______________. If the rotation is clockwise, the direction is _______________.

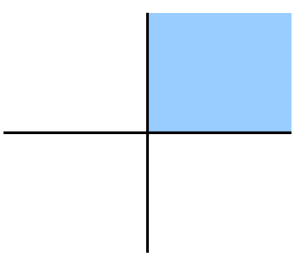
An angle in a coordinate plane is in __________________ position if:

a) its vertex is at the _____________; and
b) its initial side is the __________________ x-axis.

Since a full rotation or one revolution is _______^{0}, a measure of 1 _________ is equivalent to _______of a revolution. Common rotations of $\frac{1}{4}, \frac{1}{6}, \frac{1}{8},\ and\ \frac{1}{12}$ are used and translate into angle measures of ______^{0}, ______^{0}, _____^{0} and ______^{0} respectively. Angles in standard position can have positive and __________________ measures.

Recall: The basic trigonometric ratios: sin θ = cos θ = tan θ =

1) **Quadrant I** **$\theta < 90^0$** **$x > 0$** **$y > 0$**

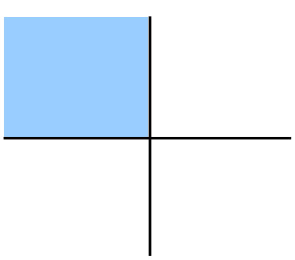


sin θ =

cos θ =

tan θ =

2) **Quadrant II** **$90^0 < \theta < 180^0$** **$x < 0$** **$y > 0$**



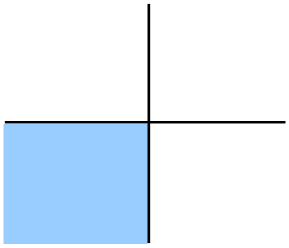
sin θ =

cos θ =

tan θ =

Word bank: counter
degree initial
negative negative
origin positive
positive standard
terminal vertex

3) **Quadrant III** <u>$180^0 < \theta < 270^0$</u> $x < 0$ $y < 0$

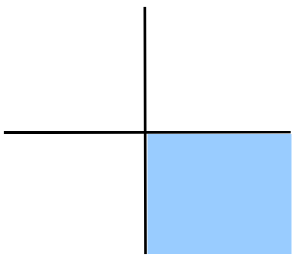


sin θ =

cos θ =

tan θ =

4) **Quadrant IV** <u>$270^0 < \theta < 360^0$</u> $x > 0$ $y < 0$

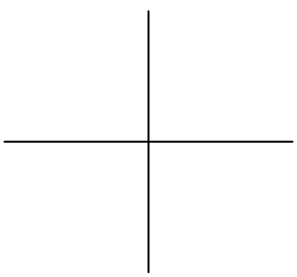


sin θ =

cos θ =

tan θ =

What happens when $\theta = 90^0$?

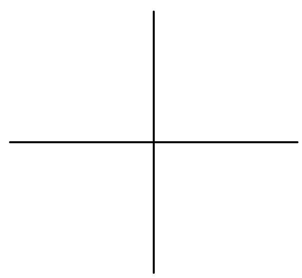


sin θ =

cos θ =

tan θ =

What happens when $\theta = 180^0$?

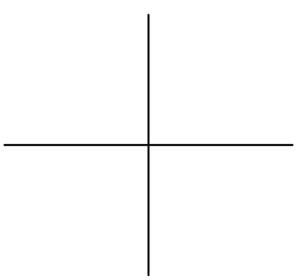


sin θ =

cos θ =

tan θ =

What happens when $\theta = 270^0$?

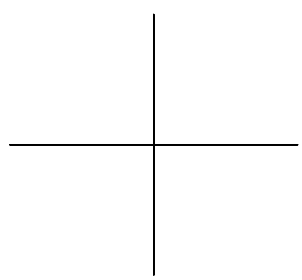


sin θ =

cos θ =

tan θ =

What happens when $\theta = 360^0$?



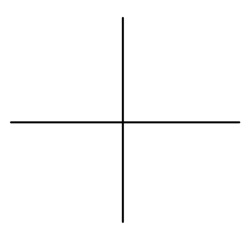
sin θ =

cos θ =

tan θ =

Note that each function is ____________________ in ______ quadrants
and ______________________ in ______ quadrants.

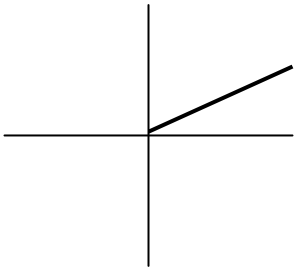
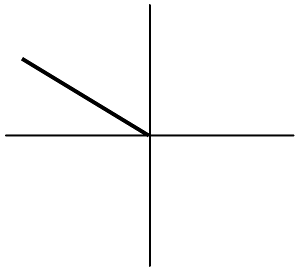
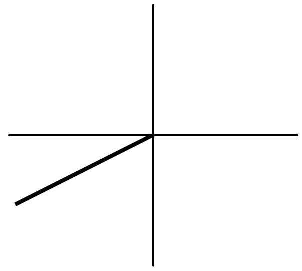
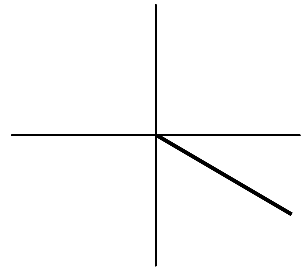
There is a _________ that will remind you which
trig function is ____________________ in each quadrant:



Word bank:
acute
nearest
negative
positive
positive
rule
terminal

Related or Reference Angle:

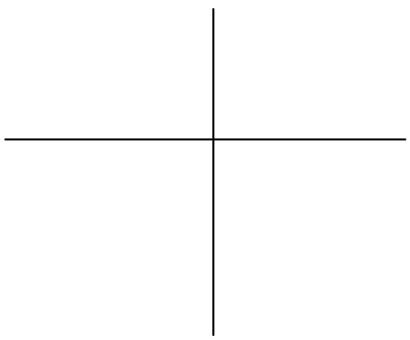
This is the ____________ angle (θ ____ 90^0) formed between the _____________ arm of the angle and the _______________ x-axis.

QI	QII	QIII	QIV
			
θ_r =	θ_r =	θ_r =	θ_r =

Example:
Determine the exact value of sin θ, cos θ, and tan θ for the angle whose terminal arm goes through the point P(2, -5). Also, find θ.

Solution:

1) Sketch



2) Find r

3) Find sin θ, cos θ, and tan θ. (Be careful with your signs)

4) Find θ_r and θ.

Example: Given $\sin\theta = \frac{2}{\sqrt{7}}$ and θ is in quadrant II, determine the value of cos θ, tan θ, θ and θ_r .

Practice:

1) Determine the *reference* (related) angle for each angle given in standard position.
   a) 98^0 b) 120^0 c) 352^0 d) 263^0 e) 75^0 f) 195^0

2) The following points are on the terminal arm of an angle. For each:
   i) Sketch the angle, θ and the reference angle, θ_r.
   ii) Determine the *exact* distance, r, from the origin to the point, P.
   iii) Determine the *exact* value of sin θ, cos θ, and tan θ.
   iv) Determine the value of θ and θ_r, to the nearest degree.

   a) P(5, 3) b) P(-3, 4) c) P(8, -2) d) P(-3, -7)

3) Point P(x, y) is on the terminal arm of each angle below in standard position. The distance between r and P is given. Determine the *coordinates* of point P, to the nearest tenth, and show a simple *sketch* of all of this information for each point.
   a) $\theta = 35^0$; r = 12 b) $\theta = 130^0$; r = 6 c) $\theta = 290^0$; r = 9 d) $\theta = 223^0$; r = 14

4) Choose any angle, θ, in standard position from quadrant I. Determine the value of: $\mathbf{\sin^2\theta + \cos^2\theta = ?}$
   Repeat this for any angle in QII. Then QIII and QIV. What do you notice? ☺

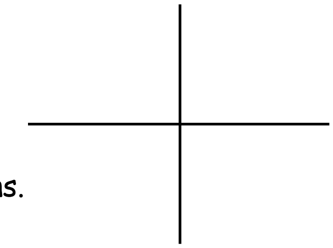
5) For what values of θ, in the interval $[0, 360^0]$ are the following conditions met?
   a) $\sin\theta < 0$ and $\cos\theta > 0$ b) $\sin\theta > 0$ and $\cos\theta \le 0$ c) $\tan\theta > 0$ d) $\cos\theta \le 0$

6) Given $\cos\theta = \frac{5}{13}$ and θ is in quadrant IV, find sinθ, tanθ, θ and θ_r .

7) Given $\sin\theta = -\frac{2}{\sqrt{5}}$ and θ is in quadrant III, find cosθ, tanθ, θ and θ_r .

Answers: 1) $82^0, 60^0, 8^0, 83^0, 75^0, 15^0$ 2) a) $r = \sqrt{34}, \sin\theta = \frac{3\sqrt{34}}{34}, \cos\theta = \frac{5\sqrt{34}}{34}, \tan\theta = \frac{3}{5}, \theta = \theta_r = 31^0$ b) $r = 5, \sin\theta = \frac{4}{5}, \cos\theta = -\frac{3}{5}, \tan\theta = -\frac{4}{3}, \theta = 127^0, \theta_r = 53^0$
c) $r = 2\sqrt{17}, \sin\theta = -\frac{\sqrt{17}}{17}, \cos\theta = \frac{4\sqrt{17}}{17}, \tan\theta = -\frac{1}{4}, \theta = 346^0, \theta_r = 14^0$ d) $r = \sqrt{58}, \sin\theta = -\frac{7\sqrt{58}}{58}, \cos\theta = -\frac{3\sqrt{58}}{58}, \tan\theta = \frac{7}{3}, \theta = 247^0, \theta_r = 67^0$
3) a) (9.8, 6.9) b) (-3.9, 4.6) c) (3.1, -8.5) d) (-10.2, -9.5) 4) Juan ☺ 5) (270, 360), [90, 180), (0, 90)U(180, 270), [90, 270]
6) $-\frac{12}{13}, -\frac{12}{5}$, $293^0, 67^0$ 7) $-\frac{1}{\sqrt{5}}$, 2, $243^0, 63^0$

Lesson 2: Trigonometric Equations



Recall:

- The four quadrants and the _______ rule for _____________ trig functions.
- Related angle, θ_r (angle between ___________________ arm & _____________ x-axis.)

Examples: Solve the following equations for θ, where $0^0 \leq \theta \leq 360^0$ or $[0^0, 360^0]$

1) $\sin\theta = 0.5$

2) $\cos\theta = 0.3$

3) $2\sin\theta + 1 = 0$

4) $\tan\theta = \sqrt{3}$

5) $3\tan\theta - 5 = 7\tan\theta - 6$

6) $4\sin\theta - 1 = 4$

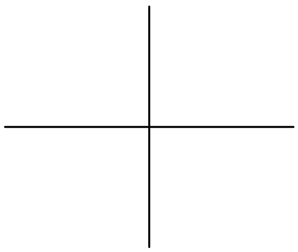
7) $\cos\theta - 3 = 0$

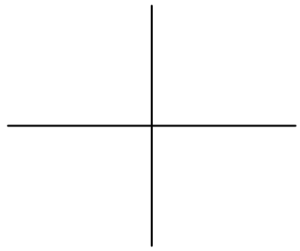
Practice: Solve the following equations for θ for the interval $[0^0, 360^0]$

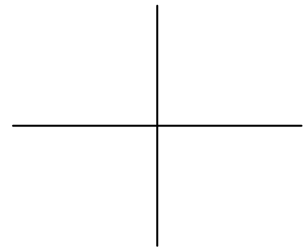
1) $-3 \sin \theta = 2$

2) $5 \cos \theta + 2 = 4$

3) $\frac{\tan\theta}{6} - 1 = 0$







<u>Practice</u>:

Determine the solution for each of the following trigonometric equations over the interval $[0^0, 360^0]$, to the nearest degree.

1) a) $\cos\theta = -\frac{2}{3}$ b) $\sin \theta + 1 = 0$ c) $\tan \theta - 2 = 5$

2) a) $2 \cos \theta = 2$ b) $-3 \sin \theta = 2$ c) $\frac{\tan\theta}{2} = 5$

3) a) $3 \cos \theta - 2 = 0$ b) $5 \tan \theta + 4 = 0$ c) $\frac{\tan\theta}{6} - 1 = 0$

4) a) $2 \cos \theta + 1 = \frac{1}{2}$ b) $4 \tan \theta - 7 = 5 \tan \theta - 6$ c) $3 \sin \theta = 4$

Answers: 1) a) 132^0, 228^0 b) 270^0 c) 82^0, 262^0 2) a) 0^0, 360^0 b) 222^0, 318^0 c) 84^0, 264^0
3) a) 48^0, 312^0 b) 141^0, 321^0 c) 81^0, 261^0 4) a) 104^0, 256^0 b) 135^0, 315^0 c) ☺

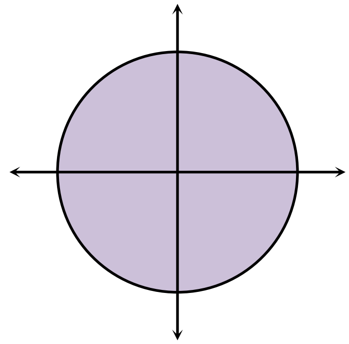
Lesson 3: Special Angles & Trigonometric Functions

Recall the three ________________ trigonometric functions:

__________ (sin)
sin θ =

_______________ (cos)
cos θ =

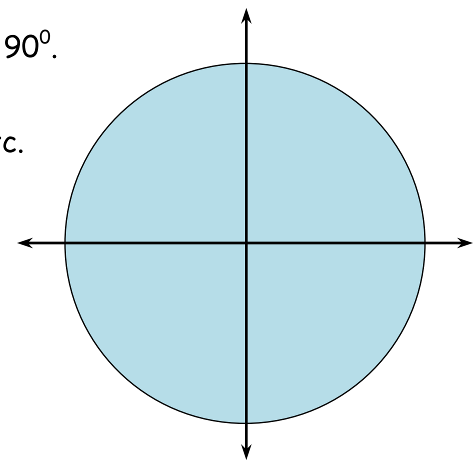
_______________ (tan)
tan θ =



Special Angles:

90^0 Family ⇒ This family contains all ________________ of 90^0.

⇒ ______________________________________, etc.



All points with an angle of **90^0**: $\cos 90^0$ =_____ $\sin 90^0$ = _____
(Along the ______________ y-axis)

All points with an angle of **180^0**: $\cos 180^0$ =____ $\sin 180^0$ = ____
(along the ______________ x-axis)

All points with an angle of **270^0**: $\cos 270^0$ =____ $\sin 270^0$ = ____
(along the ______________ y-axis)

All points with an angle of **360^0** or **0^0**: $\cos 90^0$ =_____ $\sin 90^0$ = _____
(along the ______________ x-axis)

Given the point (4, 0), what is θ ? ______ Given the point (0, 5), what is θ? __________

Given the point (0, -7), what is θ? ______ Given the point (-45.6, 0), what is θ? ________

$\sin 270^0$ = ________ $\cos 180^0$ = ________

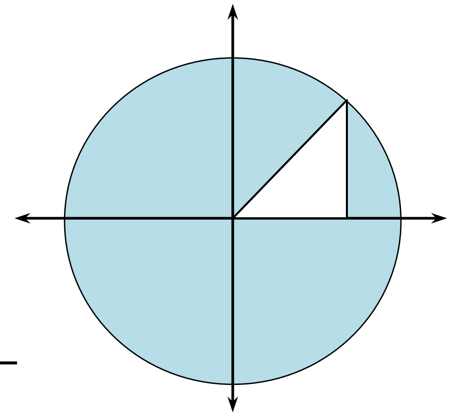
$\tan 90^0$ = ________ $\tan 180^0$ = ________ $\tan 270^0$ = ________

Within the 90^0 family of angles, the three possible values for sinθ & cosθ: ____, ____, and _____.

45^0 Family

⇒ This family contains the *multiples* of 45^0

⇒ ____, etc.



All points with an angle of 45^0: $\cos 45^0$ =_____ $\sin 45^0$ = _____

All points with an angle of 135^0: $\cos 135^0$ =_____ $\sin 135^0$ = ________

All points with an angle of 225^0: $\cos 225^0$ =______ $\sin 225^0$ = _______

All points with an angle of 315^0: $\cos 315^0$ =_______ $\sin 315^0$ = _______

Given the point (8, 8) what is θ? ________

Given the point (-6, -6), what is θ? ______

Given the point (7, -7), what is θ? _______

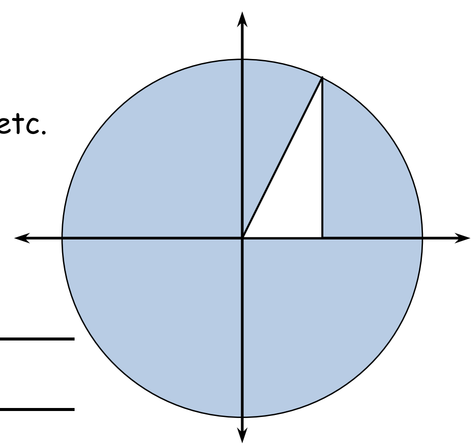
$\sin(315^0)$ = ________ $\cos(225^0)$ = ________ $\tan(45)$ = __________

$\tan(135^0)$ = ________ $\cos(45^0)$ = _________

60^0 Family

⇒ This family contains the *multiples* of 60^0

⇒ ____, etc.



All points with an angle of 60^0: $\cos 60^0$ =_____ $\sin 60^0$ = _____

All points with an angle of 120^0: $\cos 120^0$ =_____ $\sin 120^0$ = ________

All points with an angle of 240^0: $\cos 240^0$ =______ $\sin 240^0$ = _______

All points with an angle of 300^0: $\cos 300^0$ =_______ $\sin 300^0$ = ______

$\sin(120^0)$ = ________ $\cos(60^0)$ = ________

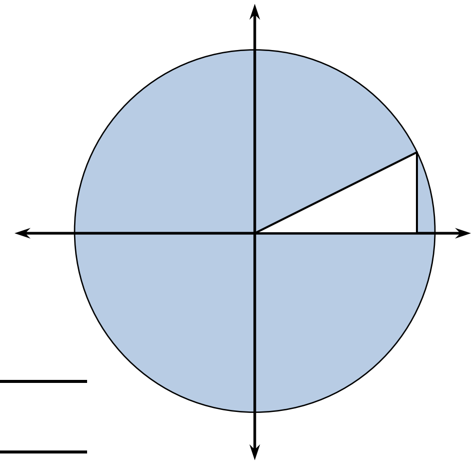
$\cos(300^0)$ = _______ $\sin(240^0)$ = _______

$\tan(60^0)$ = ________ $\tan(300^0)$ = _______

30^0 Family

⇒ This family contains the *multiples* of 30^0

⇒ ________________________________, etc.



All points with an angle of 30^0: $\cos 30^0$ =_____ $\sin 30^0$ = _____

All points with an angle of 150^0: $\cos 150^0$ =_____ $\sin 150^0$ = ________

All points with an angle of 210^0: $\cos 210^0$ =______ $\sin 210^0$ = _______

All points with an angle of 330^0: $\cos 330^0$ =______ $\sin 330^0$ = ______

$\cos(30^0)$ = ________ $\sin(150^0)$ = ________

$\sin(330^0)$ = _______ $\cos(210^0)$ = _______

$\tan(30^0)$ = ______________ $\tan(150^0)$ = _____________ $\tan(210^0)$ = _____________

When dealing with the unit circle we are easily able to determine the ___________ values of the trigonometric functions. When the **exact value** is requested, _____ ***CALCULATOR*** may be used!

Example: Find the exact value of:

$\cos(300^0) + \sin(30^0) - \cos^2(180^0)$

Example: Solve for θ

for $0^0 \le \theta \le 360^0$: $\cos\theta = -\frac{1}{2}$

Practice:

Word bank:	cosine	exact	multiples	NO	primary	sine	tangent

1) Given that $0 \le \theta \le 360^0$, determine the quadrants in which P(θ) lies.
   a) 120^0 b) 330^0 c) -45^0 d) 200^0

2) Find the exact value for each of the following (without looking at the previous notes ☺):
   a) $\sin 30^0$ b) $\sin 240^0$ c) $\cos 180^0$ d) $\tan 315^0$ e) $\cos 210^0$

   f) $\cos 45^0$ g) $\sin 300^0$ h) $\tan 330^0$ i) $\cos 120^0$ j) $\tan 225^0$

3) Find the exact value of the following:
   a) $\sin 90^0 \times \cos 360^0 \times \tan 30^0$ b) $\tan 120^0 \bullet \cos 135^0 + \sin 270^0 \bullet \tan 150^2$

4) Determine the quadrant(s) in which the point P(θ) will lie under the following conditions:
   a) $\sin\theta$ is positive.   b) $\tan\theta < 0$ c) $\sin\theta > 0$ and $\cos\theta < 0$ d) $\sin\theta = -3/5$ and $\cos\theta = 4/5$

5) True or false? $\dfrac{\sin 60^0}{1+\cos 60^0} = \dfrac{1-\cos 60^0}{\sin 60^0}$

   Justify your answer using two different methods.

6) Prove this to be true:

$$2\cos^2\left(30^0\right) - 1 = \cos^2\left(30^0\right) - \sin^2\left(30^0\right)$$

6) Determine the exact distance of each point from the origin.
   a) (4, 6)   b) (-7, 3)   c) (5, -8)

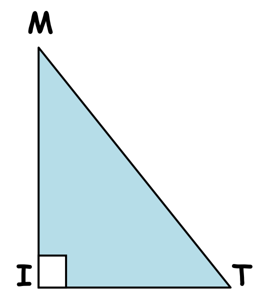
Answers: **1)** II, IV, IV, III   **2)** $\frac{1}{2}, -\frac{\sqrt{3}}{2}, -1, -1, -\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{3}, -\frac{1}{2}, 1$ **3)** $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, $\frac{\sqrt{6}}{2} + \frac{\sqrt{3}}{3}$

4) I&II, II&IV, II, IV   **5)** ☺ Use special values & algebra   **6)** $\sqrt{52} = 2\sqrt{13}$, $\sqrt{58}$, $\sqrt{89}$

Lesson 4: Trigonometry - Two Right-Triangles

Recall: the three Trigonometric ratios:

For Example: With respect to Δ MIT, identify each of these parts.



a) the side opposite angle **T**

b) the side adjacent to ∠**T**

c) the side adjacent to ∠**M**

d) the hypotenuse

e) cos **T**

f) tan **T**

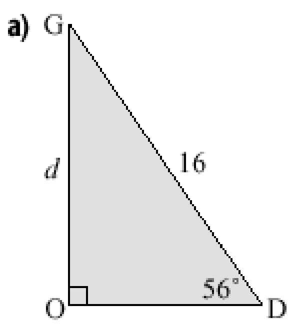
g) sin **M**

h) cos M

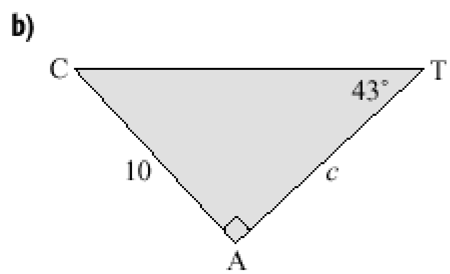
i) Pythagorean theorem

Example: Determine the missing measures in each right triangle. Round side measures to *one decimal* place and angle measures to the nearest *degree*.

∠G, d, g = ? (34°, 13.3, 8.9)

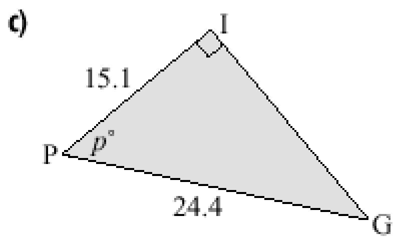


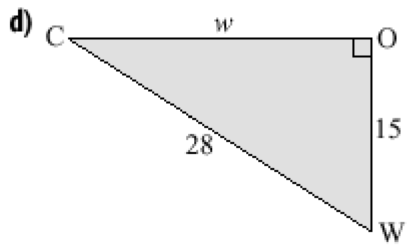
∠C, a, c = ? (47°, 10.7, 14.7)



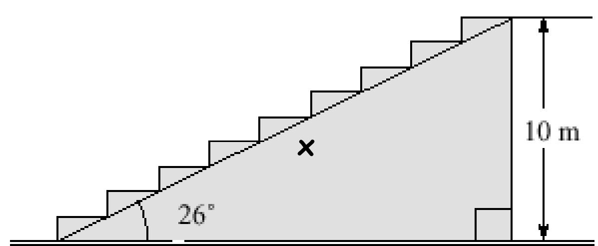
Example: Determine the missing measures in each right triangle. Round side measures to one decimal place and angle measures to the nearest degree.

$\angle P$, $\angle G$, p = ? (52^0, 38^0, 19.2) $\angle C$, $\angle W$, w = ? (32^0, 58^0, 23.6)

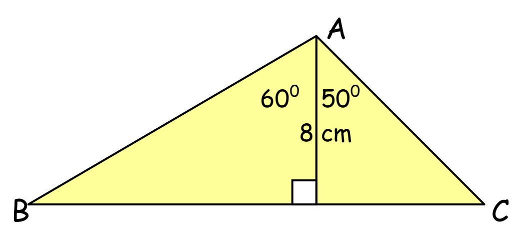




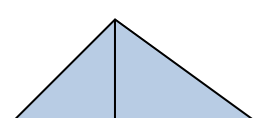
Example: The vertical distance between floors at a department store is 10 m. An escalator that has an angle of inclination of 26° connects two floors. How long (x) is the escalator? [22.8 m]



Example: Calculate the length of side BC. [23.4 m]



Example: A non-standard roof on a house has one side 18 m long and the other side is 23 m long. The peak is 14 m high.



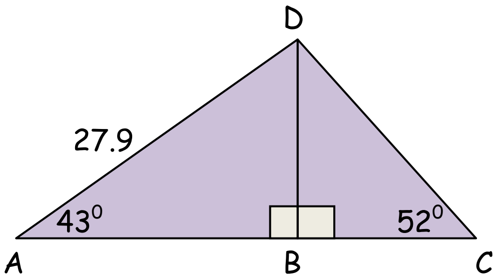
a) What is the measure of the angle formed at the peak? [52.5⁰]

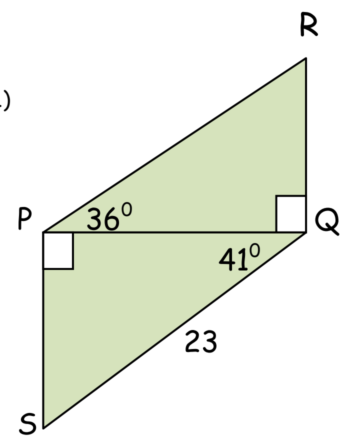
18 m 14 m 23 m

b) What is the span of the roof? [29.6 m]

Practice: (Answer #1-5 on a separate piece of paper)

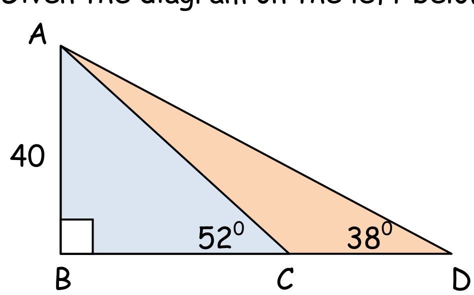
1) Given the diagram on the left below, find the length of DC. (24.1)

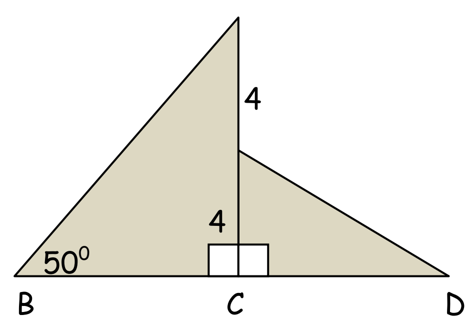




2) Given the diagram on the right above, find the length of RQ. (12.6)

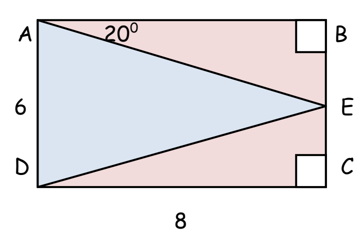
3) Given the diagram on the left below, find CD. (19.94)





4) Using the diagram on the right above, BC = CD, AE = EC = 4, ∠B = 50^0. Find ∠D. (30.8⁰)

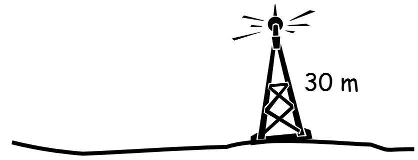
5) Given the diagram as marked, find ∠EDC. (Diagram is *not to scale*). (21.1⁰)



6) A television tower is 30 m high.

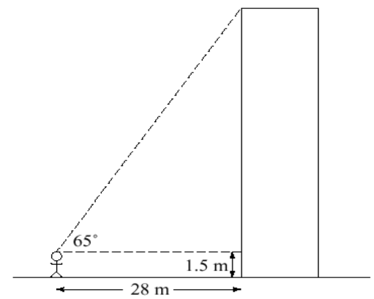
a) How long is the shadow when the sun is at an

angle of elevation of 60°? [17.3 m]



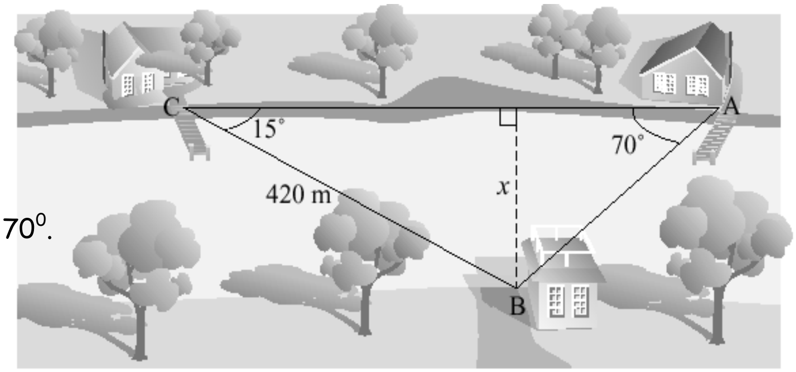
b) How long is the shadow when the sun is at an angle of elevation of 45°? [30 m]

7) At a point 28 m from a building, the angle of elevation to the top of the building is 65°. The observer's eyes are 1.5 m above the ground. How tall is the building? [61.6 m]



8)

Two cabins, A and C are located a distance apart on the bank of a river. On the other side of the river from the two cabins is a boathouse, B. It is 420 m from the cabin C to the boathouse, and the angle at C between the boathouse and cabin A is 15^0. From cabin A, the angle between C and the boathouse B is 70^0.



a) What is x? [108.7 m]

b) How far is cabin A from the boathouse? [115.7 m]

c) How far apart are the cabins? [445.3 m]

Lesson 5: Trigonometry – *Sine Law*

① When solving triangles, ALWAYS look for ____________ angles first. If right angles are involved you may use basic trigonometry (SOH ____________ ________) and/or the __________________ Theorem (_________+____=____).

② If right angles are **NOT** present, then the ____________ Law may be used.

- The Sine Law states that the ratio of the _____________ of an angle to the _______________ of the side *opposite* that angle is ______________________ for the triangle. The _____________ of this ratio is also true.

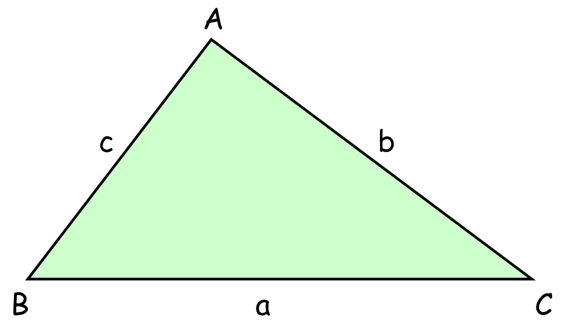
$$\frac{\sin A}{a} =$$

or

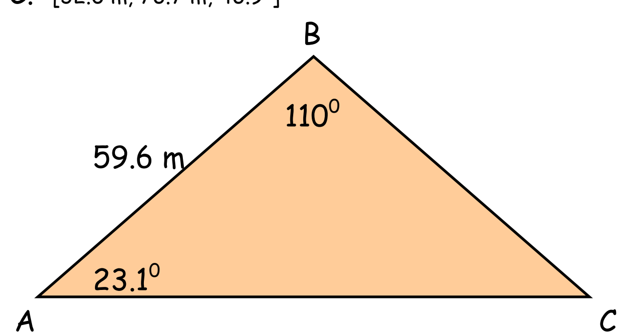
$$\frac{a}{\sin A} =$$

Where, side a is ____________________ angle A,
side b is *opposite* angle ____, etc.

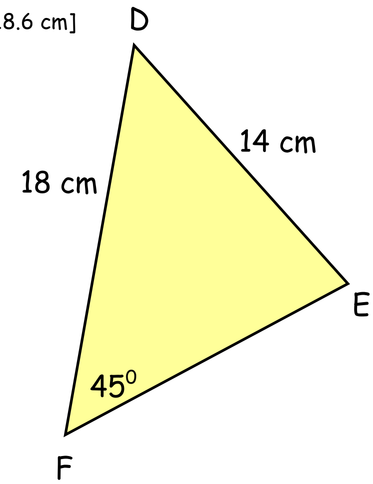
Word bank: B, CAH, constant, length, opposite, Pythagorean, reciprocal, right, Sine, sine, TOA



Example 1: Find the measures of side a and b and angle C. [32.0 m, 76.7 m, 46.9^0]



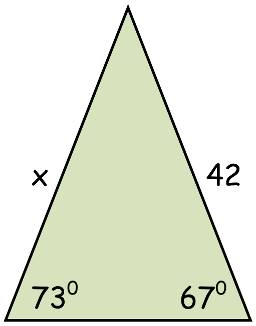
Example 2: Find the measures of angles D and E and side d. [69.6^0, 65.4^0, 18.6 cm]



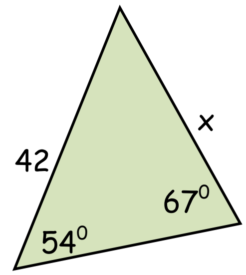
Example 3: Boats are anchored at positions J, K and M on a lake. Boats J and K are 80 m apart and J and M are 110 m apart. The angle between the lines of sight from K to J, and K to M is 120^0. What is the angle between the lines of sight from J to K and J to M? How far is it from K to M? [J=21.0^0, j=45.5 m]
Sketch?

Practice:

1) a) Find the value of x.



b) Find the value of x.



2) Side a = 5.4, $\angle B = 44^0$, $\angle C = 71^0$. Find side b.

3) $\angle A = 64.28^0$, $\angle B = 38.93^0$, c = 18. Find a.

4) b = 67, c = 67, $\angle B = 59^0$. Find $\angle A$, where ΔABC is acute.

5) a = 32, b = 52, $\angle A = 33^0$. Find $\angle B$, where ΔABC is acute.

Answers: 1) a) 40 **b)** 37 **2)** 4.1 **3)** 17 **4)** 62^0 **5)** 62^0

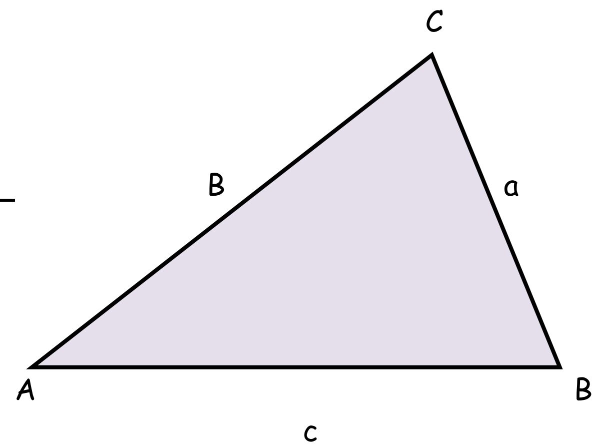
Lesson 6: Trigonometry – *Cosine Law*

If we draw a perpendicular line from AB to C and apply the ________________ Theorem, we get

h^2 = ____________ and h^2 = ____________

If we equate the h^2's: $h^2 = h^2$

$a^2 - x^2 = b^2 - (c - x)^2$



$$a^2 = b^2 + c^2 - 2bc \cos A \quad \Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

★ When solving triangles:

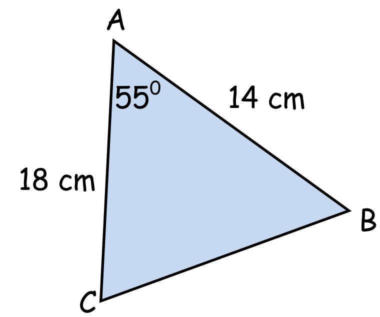
① Check for right- ________ (90^0). ➔ _________ ***trig***onometric ratio's (SOH CAH TOA)

② Check for ________________ (a side and an _______________ angle). ➔ _______ ***law***.

③ If *none* of the above possibilities exist: ➔ _____________ ***law.***

Example: Re-write the cosine law for *side* t and ∠T of △ STR.

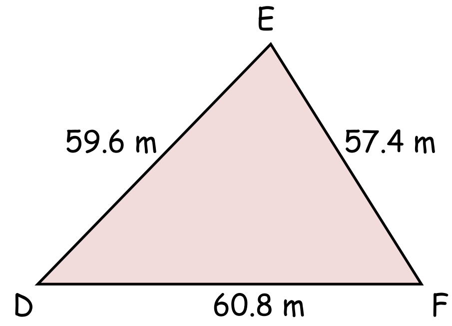
Example: Find the measure of side **a** and angles **B** and ***C***. [15.2 cm, 76^0, 49^0]



1. Right angles?
2. Sine-Law ratio's? (opposites?)
3. Use _______________ Law

Notice the relationship between the smallest angle and the ___________________ side. Also the largest angle and the ________________ side.

Example: Find the measures of angles **D**, **E** and **F**. [56.9^0, 62.5^0, 60.6^0]



Practice:

1) a = 15, b = 19, $\angle C = 50^0$. Find c.

2) c = 13, b = 18, $\angle A = 70^0$. Find a.

3) b = 42, c = 37, $\angle A = 150^0$. Find a.

4) $a = 15$, $b = 10$, $c = 12$. Find $\angle A$ to the nearest degree.

5) $a = 5$, $b = 3$, $c = 7$. Find $\angle B$ to the nearest degree.

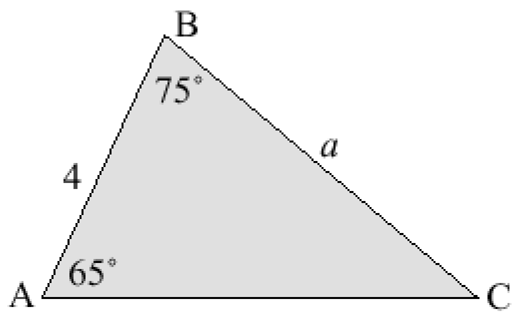
6) $a = 8$, $b = 11$, $c = 6$. Which is the *smallest* angle of $\triangle ABC$? Find it to the nearest degree.

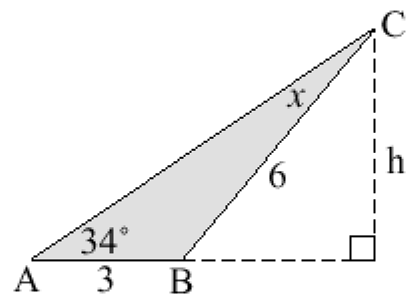
Answers: **1)** 14.8 **2)** 18.2 **3)** 76.3 **4)** 85^0 **5)** 22^0 **6)** $\angle B=32^0$

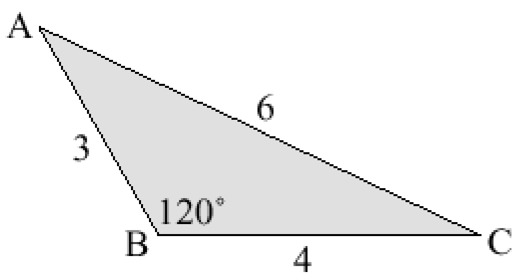
Lesson 7: Applications of Sine & Cosine Laws

Diagrams are NTS (*not* to scale). If no diagram is given, _______________ one to represent the situation before completing the exercise. Express all *lengths* to the nearest *tenth* and all *angles* to the nearest *degree*.

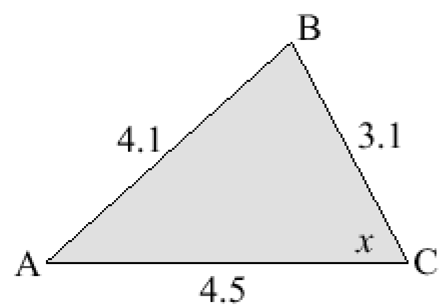
1. For each triangle, determine the indicated measures. [5.6, 4.6, 6.1, 62.1^0]

a) 

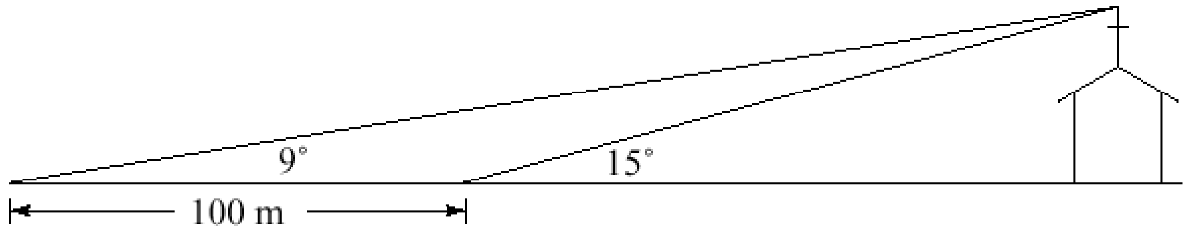
b) 

c) 

Angle B = ?

d) 

2. From a certain point, the angle of elevation to the top of a church steeple is 9°. At a point 100 m closer to the steeple, the angle of elevation is 15°. Calculate the height of the steeple. [38.7 m]



3. A tower is supported by two guy wires attached to the top of the tower and fixed to the ground on opposite sides of the tower 27 m apart. One wire is 19.3 m long and meets the ground at an angle of 53°. [15.4 m, 21.8 m, 45^0]

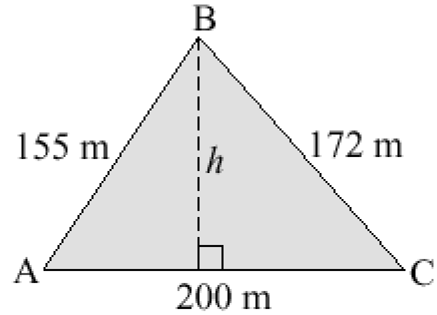
a) What is the height of the tower?

b) What is the length of the second wire?

c) What angle does the second wire make with the ground?

4. A triangular park has sides of length 200 m, 155 m and 172 m. [56.3^0, 129.0 m, 12 900 m^2]

a) Determine Angle A.



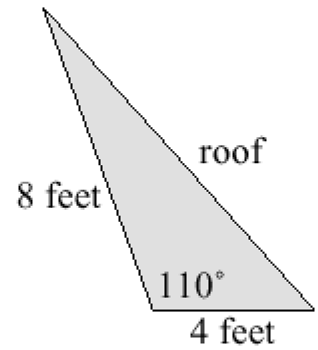
b) Determine h.

c) Calculate the area of the park.

5. To determine the height of a cliff, a surveyor measured the angle of elevation of the top of the cliff from a point away from the base to be 45°. He then moved 20 m further away from the base of the cliff and found the angle of elevation to the top to be 37°. Determine the height of the cliff. [61.2 m]

6. The end of a lean-to for cattle is in the shape of an obtuse triangle as shown below. [10.1 ft., 48.1°]

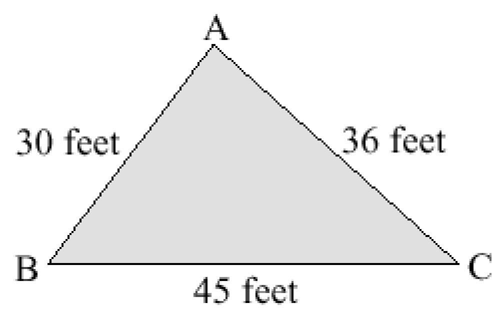
a) Determine the length of the roof.



b) Determine the angle that the roof of the shed makes with the ground.

7. In the design of a ski chalet, the slant of the roof must be steep enough for the snow to slide off. An architect originally designed the roof to span 45 feet with slanted sides of 36 ft and 30 ft. He decided it would be better to modify the roof by increasing the measure of the smaller angle by 10° thus increasing the length of the side opposite that angle. [51.6°, 36.2 ft.]

a) What is the new angle measure?



b) What is the new length of this side?

Lesson 8: Trigonometry - Ambiguous Case Problems

Recall basic trig. ratio's: ***sin*** θ **=** ***cos*** θ **=** ***tan*** θ **=**

Sine Law: Cosine Law:

When solving triangles and you are given ______ sides and the non-included ___________ (SSA), the triangle may not be _________________. It is possible that ______ triangles, ___ triangle, or _______ triangles may exist for the given measurements. This is called the _________________________ case.

The __________________ case occurs when the angle is ______________ the __________________ of the two sides. However, if the given angle is opposite the __________________ of the two given sides, there is __________ ambiguity. That is, only ________ triangle exists.

Recall: (non-ambiguous cases)

- If given __________ or ________ use _____________ Law. ⇒ only 1 Δ exists.

- If given __________ or ________ use _____________ Law. ⇒ only 1 Δ exists.

Investigation: For each of the following 5 cases:

a) Draw the triangles *to scale*; and

b) Determine *how many* triangles may be formed: **0**, **1** or **2**.

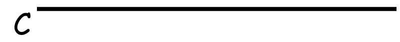
Case 1: ΔABC with $\angle C = 100^0$, b = 5 cm, c = 4 cm

______ Δ's

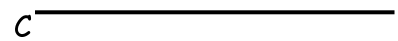
C

Case 2: ΔABC with $\angle C = 30^0$, b = 5 cm, c = 2 cm

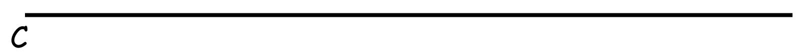
______ Δ's



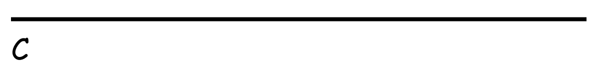
Case 3: ∆ABC with ∠C = 30^0, b = 5 cm, c = 2.5 cm
______ ∆



Case 4: ∆ABC with ∠C = 30^0, b = 5 cm, c = 6 cm
______ ∆



Case 5: ∆ABC with ∠C = 30^0, b = 5 cm, c = 3 cm
______ ∆'s (solve these)



Practice:

1) In $\triangle ABC$, $a = 4$, $b = 6$ and $\angle A = 30^0$. Solve the $\triangle$.

A

2) In $\triangle DEF$, $e = 2.5$, $f = 5$ and $\angle E = 30^0$. Solve the $\triangle$.

3) In ΔGHK, $h = 2$, $f = 10$ and $\angle H = 40^0$. Solve the Δ.

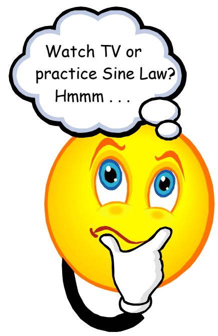
4) In ΔMNP, $m = 6$, $n = 9$ and $\angle M = 36^0$. Solve the Δ.

5) In ΔABC, $a = 1.9$, $b = 6.1$, $\angle A = 31^0$. Solve the Δ.

6) In ΔDEF, $d = 3$, $e = 6$, $\angle D = 30^0$. Solve the Δ.

7) In ΔGHI, $g = 4$, $h = 3$, $\angle H = 29^0$. Solve the Δ.

8) In ΔJKM, $j = 8.8$, $k = 12$, $\angle J = 27^0$. Solve the Δ.



Do you still want *more* practice?? Here you go . . .

Find the possible values of the indicated *side*:

9) In ΔABC, $\angle B = 34^0$, $a = 4$, $b = 3$. Find c.

10) In ΔXYZ, $\angle X = 13^0$, $x = 12$, $y = 15$. Find z.

11) In ΔABC, $\angle B = 34^0$, $a = 4$, $b = 5$. Find c.

12) In ΔRST, $\angle R = 130^0$, $r = 20$, $t = 16$. Find s.

13) In ΔMBT, $\angle M = 170^0$, $m = 19$, $t = 11$. Find b.

14) In ΔABC, $\angle B = 34^0$, $a = 4$, $b = 2$. Find c.

Find all the possible values of the indicated *angle*.

15) In ΔABC, $\angle A = 19^0$, $a = 25$, $c = 30$. Find $\angle C$.

16) In ΔHDJ, $\angle H = 28^0$, $h = 50$, $d = 20$. Find $\angle D$.

17) In ΔXYZ, $\angle X = 58^0$, $x = 9.3$, $z = 7.5$. Find $\angle Z$.

18) In ΔBIG, $\angle B = 39^0$, $b = 900$, $g = 1000$. Find $\angle I$.

Answers: **1)** ① $B=38.7^0$, $C=111.3^0$, $c=7.45$ ② $B=141.3^0$, $C=8.7^0$, $c=1.21$ **2)** $D=60^0$, $F=90^0$, d4.33 **3)** 😐
4) ① $N=61.8^0$, $P=82.2^0$, $p=10.2$ ② $N=118.2^0$, $P=25.8^0$, $p=4.4$ **5)** 😐 **6)** ① $E=90^0$, $F=60^0$, $f=5.2$
7) ① $G=40^0$, $I=111^0$, $i=5.8$ ② $G=140^0$, $I=11^0$, $c=1.2$ **8)** ① $K=38^0$, $M=115^0$, $m=17.6$ ② $K=142^0$, $M=11^0$, $m=3.7$
9) 1.3 or 5.3 **10)** 3.1 or 26.1 **11)** 7.8 **12)** 5.5 **13)** 8.0 **14)** 😐 **15)** 23^0 or 157^0 **16)** 10.8^0 **17)** 43.1^0 **18)** 96.6^0 or 5.4^0