Unit: Polynomials: Multiplying and Factoring

Name ________________________ Dates Taught ________________

Specific Outcome			
10I.A.1	Demonstrate an understanding of factors of whole numbers by determining: • Prime factors • Greatest common factor • Least common multiple		
10I.A.3	Demonstrate an understanding of powers with integral and rational exponents		
10I.A.4	Demonstrate an understanding of the multiplication of polynomial expressions		
10I.A.5	Demonstrate an understanding of common factors and trinomial factoring		

Comments : ____

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Outcome: 10I.A.4: Multiplying Polynomials (Part 1)

Adding Polynomials: Combine like terms (add ____________________)

$(5a - 6b + 3c) + (8a + 5b - 4c) =$ ____________________

Subtracting Polynomials: Multiply the _______________ through the brackets

$(4x^2 - 2x + 3) - (3x^2 + 5x - 2) =$ ____________________

$=$ ____________________

Multiplying Polynomials (Monomial by Monomial):

1) Multiply the coefficients 2) Add the exponents

$(2x^2)(7x) =$ ____________________

$(-4a^2b)(3ab^3) =$ ____________________

Dividing Monomials: 1) Divide the coefficients 2) Subtract the exponents

$\frac{20x^3y^4}{-5x^2y^2} =$ ____________________

Multiplying Monomial by Polynomial:

$5y^2(x^2 - y) =$ ____________________

$4y(2y^2 + 3y - 1) =$ ____________________

Binomial by Binomial :

- A technique for multiplying two binomials is using the F.O.I.L. method. The letters F. O. I. L. stand for _______, ________, ________, ________
- We always <u>multiply</u> these terms.

Steps :

1) Identify the first term in each bracket and ______________ them together.
2) Identify the most outside terms of the expression and multiply them together.
3) Identify the most inside terms of the expression and multiply them together.
4) Identify the last term in each bracket and multiply them together.
5) Collect like terms.

Examples:

$(x + 2)(x + 5) =$

<u>First</u> <u>Outer</u> <u>Inner</u> <u>Last</u>

$(x + 6)(x + 8) =$ ____

= ____

$(2x - y)(3x + y) =$ ____

= ____

$(x - 2y)(x + 2y) =$ ____

= ____

Binomial Squared:

$(x + 5)^2 =$ ______________________________

= ______________________________

= ______________________________

$(2x - y)^2 =$ ______________________________

= ______________________________

Try: FOIL game --
http://homepage.mac.com/markgreenberg2/Games/MrGreenbergsGames.html

Homework: Textbook Page 87 #3-5

Outcome: 10I.A.4: Multiplying Polynomials (Part 2)

Binomial by Trinomial: Distribution Method

Example 1 Multiply:

a) $(y - 3)(y^2 - 4y + 7) =$ ______________________________

= ______________________________

= ______________________________

b) $(2x - 1)(2x^2 + 5x - 3) =$ ______________________________

= ______________________________

= ______________________________

Example 2 Expand the following:

a) $3(x - 1)(2x - 3) =$ ______________________________

____

b) $(5a + 4) + (a - 1)(a + 2) - (2a - 3) =$ ______________________________

____

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Homework: Textbook Page 87 #6 - 10 and MCAL20S: Exercise 1

Outcome: 10I.A.1 - *Prime Factors*

- When a factor of a number has exactly two divisors, one and itself, the factor is a prime factor.
- For example, the factors of 12 are 1, 2, 3, 4, 6, and 12. The prime factors of 12 are 1, 2, and 3. To determine the prime factorization of 12, write 12 as a product of its prime factors: 2 x 2 x 3, or $2^2 \times 3$

The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29

Natural numbers greater than one that are not prime, are *composite*.

Example 1: Write the prime factorization of 3300.

Method 1: Factor Tree

Method 2: Repeated Division

<u>Outcome: 10I.A.1</u> – *Least Common Multiple*

- The **least common multiple (LCM)** is the smallest multiple shared by two or more terms. To generate multiples of a number, multiply the number by the natural numbers; that is, 1, 2, 3, 4, 5, and so on.
- For example, some multiples of 26 are:
  $26 \cdot 1 = 26 \quad 26 \cdot 2 = 52 \quad 26 \cdot 3 = 78$
- For two or more natural numbers, we can determine their **least common multiple**.

Example 2: Determine the least common multiple of 15, 20, and 30.

Method 1: Listing Multiples of All Numbers

Method 2: Listing Multiples of the Largest Number (and divide by the other numbers)

Example 3: Mei is stacking toy blocks that are 12 cm tall next to blocks that are 18 cm tall. What is the shortest height at which the two stacks will be the same height?

Homework: Textbook Page 91 #2, 3, 5

Outcome: 10I.A.1 & Outcome: 10I.A.4: - *Common Factoring*

- The **greatest common factor (GCF)** is the largest factor shared by two or more terms. This is the largest number that both terms can be divided by.

For example; The factors of 12 are 1, 2, 3, 4, 6, and 12
The factors of 18 are 1, 2, 3, 6, 9, and 18

The GCD of 12 and 18 is ________

Example 1: List the factors of each of the following numbers. Then, identify the Greatest common factor.

a) 15 and 30

b) -24 and -48

Example 2: Determine the greatest common factor of $4xy$ and $2x^2y$. ________________

Example 3: Determine the greatest common factor of the following sets of terms:

$18x^2yz$, $27x^2y^2z$, $9x^2y^2$ ______________________

Common Factoring

Factoring is the ______________ process of__________________. The better you are at multiplying, the better you will be at factoring.

Multiplication →	***Factoring*** →
$5x(x - 2y) = 5x^2 - 10xy$	$9x^2 - 15x = 3x(3x - 5)$
$(x - 3)(x + 5) = x^2 + 2x - 15$	$x^2 + 8x + 15 = (x + 3)(x + 5)$

1) Common Factoring:

- When factoring, ___________ begin by looking for ___________ terms. It could be a number, a variable or both.
- Place this common factor in front of parentheses, with the remaining polynomial ___________ the parentheses.
- Once this is done, the same number of terms as in the original question should be inside. (i.e. a ______________ leaves a _______________.)

Examples: Factor the following:

i) $4x + 8 =$__________(___________)
Common factor Remaining factor

ii) $8xy - 32y^2 =$_____________________

iii) $7n^2 - 49n =$ ___________________

iv) $15w^3 + 5w =$ _____________________

iv) $b - b^2r^3c =$ ___________________

vi) $12n^3 - 16n^2 + 32n =$ ________________

vii) $3x^3 - 6x^2y + 9xy^2 =$ ____________

- Factoring can always be quickly and easily checked by _____________________ the polynomials together to see if the product is the original polynomial.

Homework: Textbook Page 91 #1, 4, 6, 7

Outcome: 10I.A.4: Trinomial Factoring

- Trinomials will factor to 2 ______________brackets.

Example: $x^2 + 5x + 6 =$

Steps:

① ALWAYS factor out any ______________ terms/variables first.

② Identify the _______ of the last term of the trinomial.

③ Next, determine which ______ of factors either _______ up to or _____________ to get the *middle* term

④ Therefore, $x^2 + 5x + 6$ factors to (_______)(_______)

Examples: Factor the following trinomials fully, if possible:

1. $x^2 + 9x + 18$

2. $y^2 - 2y - 15$

3. $5 + b^2 - 6b$

4. $a^2 - 4a - 60$

5. $x^2 + 8xy + 16y^2$

6. $p^4 - 2p^2 - 15$

7. $2x^2 + 8x + 6$

8. $x^2 - 5x - 6$

9. $3x^3 - 18x^2 + 27x$

10. $2x^2yz^3 - 10xyz^3 - 48yz^3$

Homework: Textbook Page 95 #4, 5, 8, 10

Outcome: 10I.A.4: Factoring Difference of Squares

a) Perfect Square *Binomials*: $ax^2 - by^2$

- A difference of squares has 3 main features:
  1. The first term is a perfect ___________.
  2. The second term is a ________________ square.
  3. They are separated by a ___________________ sign.

Example: $x^2 - 16y^2$

- The _____________ term is absent because it is ________________.

  Eg. $x^2 - 0xy - 16y^2$

- Factoring a perfect square *binomial* results in two similar binomials, that differ only in the _______________ sign.

- **To factor a difference of squares:**
  ① Remember to ALWAYS begin factoring by looking for a ___________ factor.
  ② The first term of the binomials' comes from the square __________ of the ____________ term.
  ③ The __________ term of the binomials' comes from the square __________of the second term.
  ④ Place a ______________ sign in one parentheses and a _______________ in the other.
  ⑤ Check the result by using F.O.I.L .

Example from above: $x^2 - 16y^2$ = (_________)(_________)

Examples:

1. $x^2 - 9$ = (__________)(__________)	**5.** $3x^3 - 48x$ = =
2. $225b^2 - a^2$ = (__________)(_________)	**6.** $x^4 - 16$ =
3. $49 + x^2$ = (_________)(__________)	
4. $-y^2 + 36$ = (________)(__________)	

B) Perfect Square *Trinomials*: $x^2 \pm bxy + cy^2$

Example:
$x^2 - 8xy + 16y^2$

- A perfect square trinomial has ________ main features:
  1. The first term is a ___________________ _______________.
  2. The ____________ term is a perfect square. The sign of the last term is always _________________.
  3. The _______________ term can be either positive or negative. It is always double the square root of the last term.
- Factoring a perfect square *trinomial* results in two ______________ _______________

Example: Factor: $x^2 - 8xy + 16y^2 =$

Check:

Examples: Factor the following trinomials fully, if possible.

1. $49 + 14x + x^2 =$

2. $5b^3 - 40b^2 + 80b =$

3. The *volume* of a *rectangular* prism is represented by $2x^3 - 24x^2 + 72x$. What are possible *dimensions* of the prism?

Homework: Textbook Page 99 #4, 5, 6 (a-e), 7 (a- g)

Outcome: 10I.A.4: Factoring $ax^2 + bx + c$ (leading coefficient) (FOIL Method)

- **Use this method anytime there is a ________________ in front of your x^2 which cannot be factored out.**

Factor: **$4x^2 - 18x - 10$**	
	① ALWAYS begin factoring by checking for common _______.
	② Determine the ____________ of the first term of the trinomial.
	③ Determine the ____________ of the last term of the trinomial.
	④ We need to find the right ____________________ of these factors that will cause the binomials to multiply out to your original trinomial. -You can check this by applying ______________. -If the product does not come out to be the given trinomial, then you need to try again.

Examples: Factor the following fully, *if possible*:

1. $2y^2 + y - 1 =$

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2. $3a^2 + 5a + 2 =$

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3. $3x^2 + 9x + 6=$

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4. $10x^4 + 8x^2 - 2 =$

5. $3b^4 - 5b^2 - 2 =$

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6. $2c^2 + 2c - 3 =$

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7. $3d^3 + 10d^2 + 8d =$

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8. $2g^2 - 13g + 15 =$

Homework: Textbook Page 95 #6, 7, 9, 11