Unit: Trigonometry

Name _____ Dates Taught _____

General Outcome			
10I.M.4	• Develop and apply the primary trigonometric ratios to solve problems that involve right triangles		

Achievement Indicators:

-Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios

-Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle.

-Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean Theorem

-Solve a problem that involves direct and indirect measurement, using measurement instruments such as a ruler, clinometer or metre stick, the trigonometric ratios, or the Pythagorean Theorem (for direct measurement, see measurement unit)

Comments : _____

_____

_____

_____

Activity 1

Ratio and Proportion in Similar Triangles

1. Draw a triangle where two of the angles are between 10° and 60°.
   a) Note: Avoid choosing a 30°-60°-90° triangle. This will be covered in later sections.
   b) Draw the base horizontally.
   c) Label the base a. Label the other two sides b and c in a clockwise direction from side a. Label angles with capitals (as shown in Diagram 1).

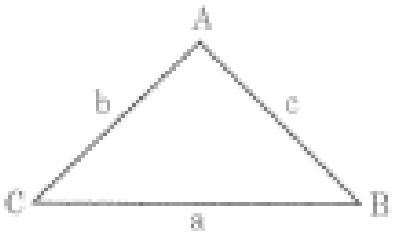


Diagram 1

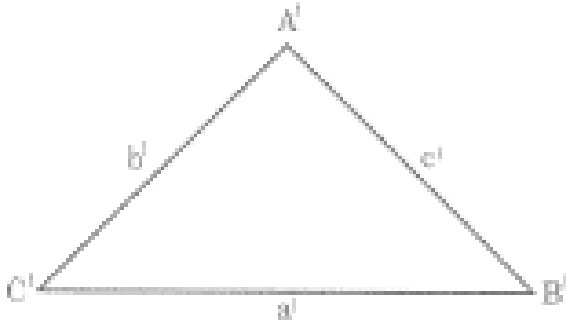
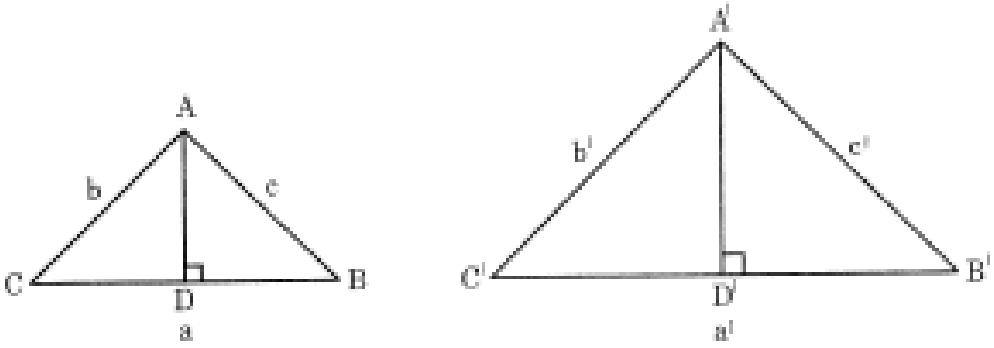


Diagram 2

2. Draw a second triangle, larger than the first one, where the three angles of the triangle are equal to the three angles of the previous triangle.
   a) Draw the base horizontally.
   b) Label the base a'. Label the other two sides b' and c' in a clockwise direction from a'. Label angles A', B', and C' (as shown in Diagram 2).

3. On the first triangle, draw a line AD that is perpendicular to BC. On the second triangle, draw a line A'D' that is perpendicular to B'C' (as shown below).



4. Fill in the data table completely.
5. Compare the angle ratios. What pattern do you see?
6. Compare the lengths of sides ratios. What patterns do you see?
7. What is the ratio of the two triangles' areas? How does this relate to the other ratios in the data table?

Check out this applet:
http://members.shaw.ca/ron.blond/SimilarTriangles.APPLET/index.html

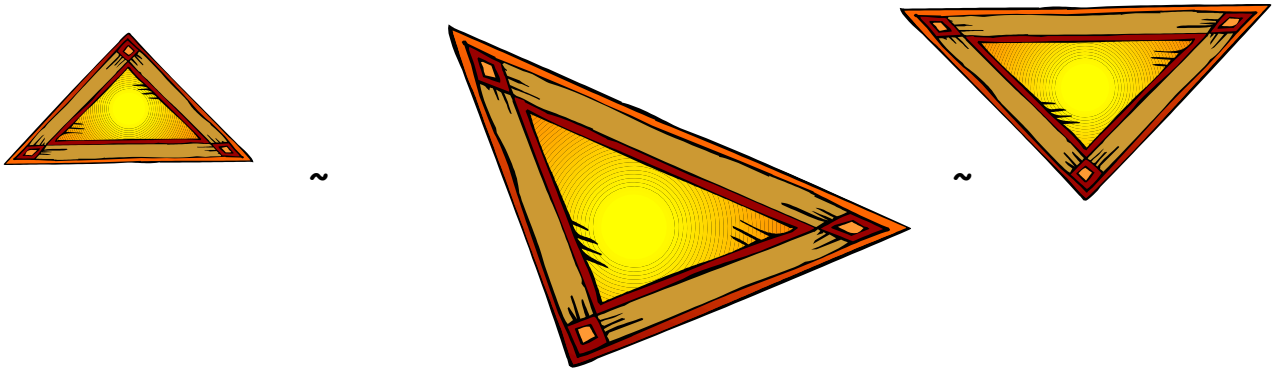
Outcome:10I.M.4 – *Similar Triangles*

Trigonometry:

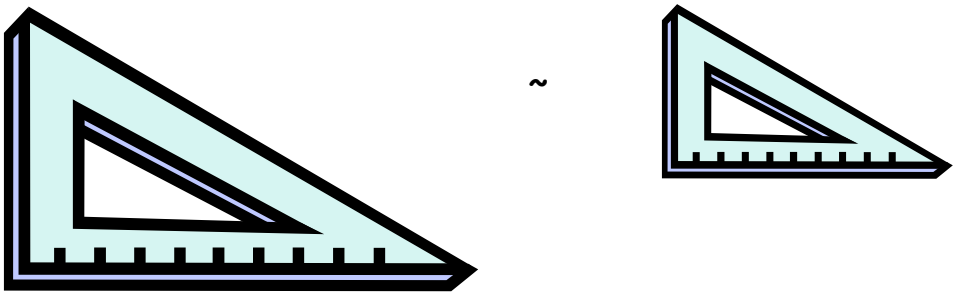
Trigonometry is a branch of mathematics that studies triangles, particularly right triangles. Trigonometry deals with relationships between the sides and the angles of triangles, and with trigonometric functions, which describe those relationships and angles in general.

From a functional point of view, "Trig" allows for the measurement of inaccessible and difficult to measure objects. It is used in surveying, construction, navigation, engineering, physics and astronomy.

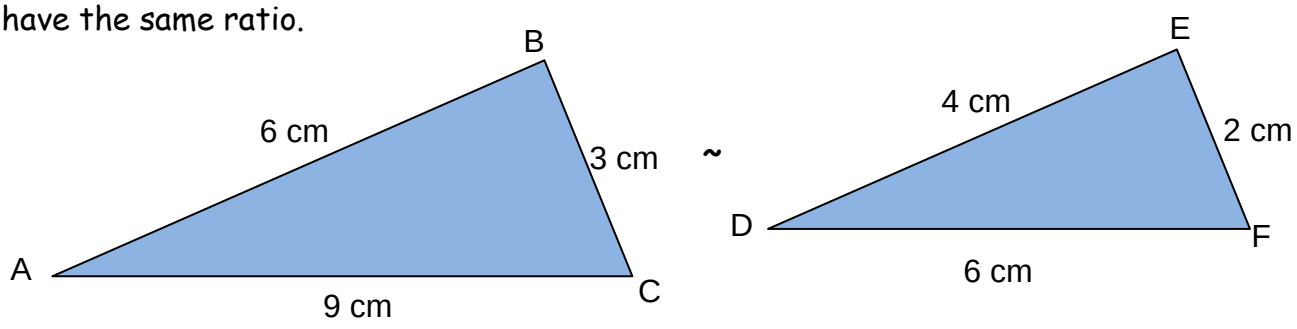
Similar Triangles: One of the tools that we use in Trigonometry is **Similar Triangles**. Similar triangles have the same general shape, but differ in size from one to another.



If you measure the sides of Similar Triangles, you will find that the corresponding sides are different lengths, but the corresponding angles are the same.



We actually say that the sides are proportional, meaning that any two corresponding sides have the same ratio.



$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{6}{4}$$

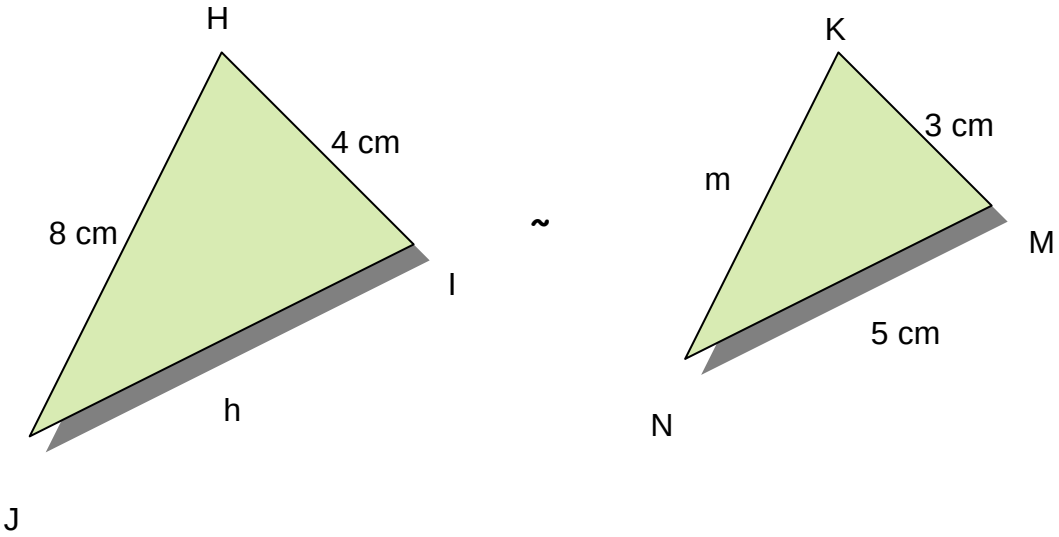
Example 1: Solve for the missing variable in each of the following proportions.

$$\frac{15}{36}=\frac{a}{60} \qquad\qquad \frac{5}{3.5}=\frac{8}{y}$$

Solution:

Note: When drawing and labeling triangles; **angles** are identified with an **upper case** letter and **sides** are identified with a **lower case** letter.

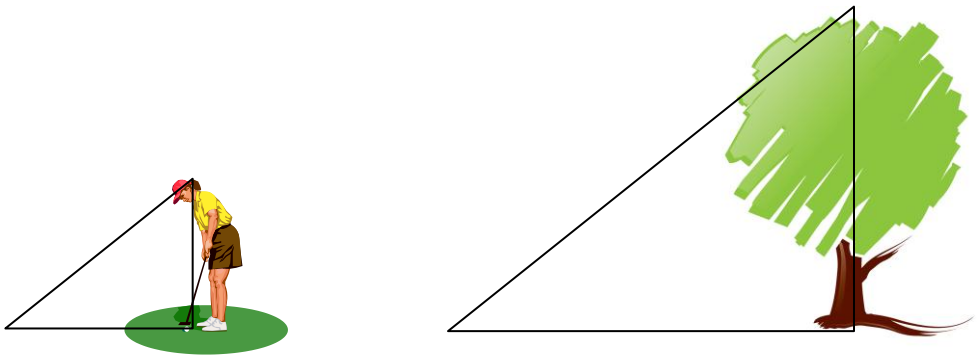
Example 2: If ΔHIJ ~ ΔKMN, find the ratio between the sides, and find the missing sides m and h.



Solution:

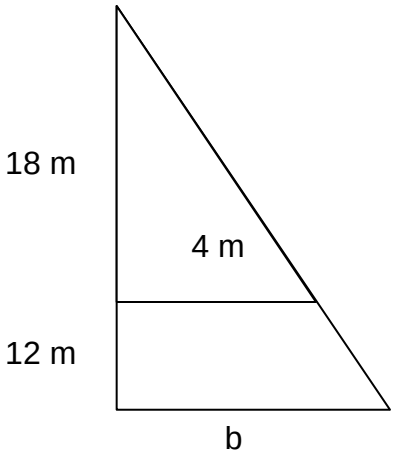
Hint: 1. Find the ratio by using the corresponding sides where both values are given
2. Place the side you are trying to find on the top row

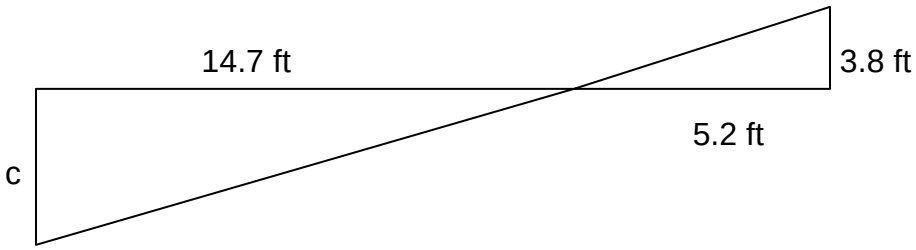
Example 3: If a tree casts a shadow 6.9 m long at the same time as a golfer whose shadow is 2.3 m long, find the height of the tree if the golfer is 1.5 m tall.



Solution:

Example 4: If the following triangles are similar, find the lengths of the missing sides.



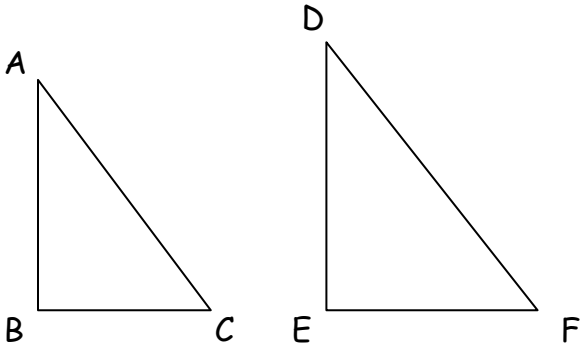


Solution:

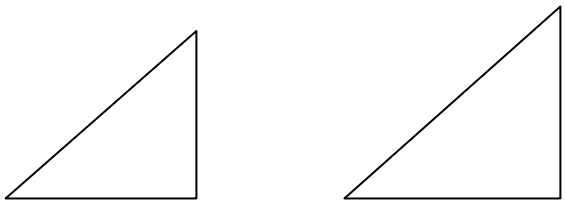
Homework:

For each set of triangles, (a) draw and label triangles with the given information; (b) determine if the triangles are similar; and (c) if they are similar, determine the missing side.

1. In triangles ABC and DEF, $\angle A = \angle D = 39.5^0$, $\angle B = \angle E = 90^0$, a = 5, c = 8.6, d = 10, f = ?.



2. In triangles GHI and JKL, $\angle G = \angle L = 90^0$, $\angle H = 45^0$, h = 6, i = 6, j = 10, $\angle K = 45^0$, k = ?



3. In triangles MNO and PQR, $\angle M = \angle P$, $\angle N = \angle Q = 90^0$, r = 0.75, p = 0.45, m = 10.5, o =?

4. In triangles STU and VWX, $\angle T = 70^0$, $\angle U = 30^0$, s = 14, u = 8, $\angle V = 60^0$, $\angle X = 40^0$, v = 30, x = ?

5. In triangles ABC and DEF, $\angle A = 54^0$, $\angle C = 67^0$, a = 37, c = 42, $\angle D = 54^0$, $\angle E = 59^0$, d = 7, f = ?

6. In triangles GHI and JKL, $\angle H = 90^0$, $\angle I = 64.9^0$, g = 5, h = 13, $\angle J = 25.1^0$, $\angle K = 90^0$, j = 10, k = ?

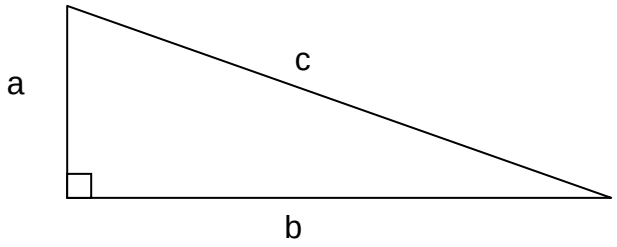
7. I am standing beside the school on a sunny day and the length of my shadow is 3.2 m. I am 1.8 m tall. The shadow that the school makes is 19.0 m long. How tall is the school?

8. I placed a mirror flat on the ground and 20 m from the school. When I stand 1.0 m beyond the mirror, I can see the top of the building in the mirror. If my eyes are 1.7 m above the ground, how tall is the school?

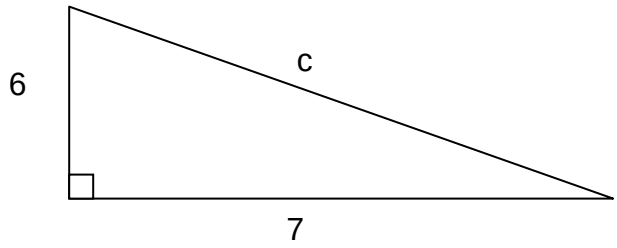
Outcome:10I.M.4 – *Pythagorean Theorem*

Pythagorean Theorem: $a^2 + b^2 = c^2$ or $c = \sqrt{a^2 + b^2}$

- This is used for finding the hypotenuse (The Hypotenuse is the diagonal line of the triangle and is labeled the letter "c") of Right Triangles given the two sides.
- **Note:** You may also want to use the formula $a = \sqrt{c^2 - b^2}$ or $b = \sqrt{c^2 - a^2}$ to find a side when given the hypotenuse.



Example: Find **c** to three decimal places.



1. Write the formula.

2. Substitute the given sides.

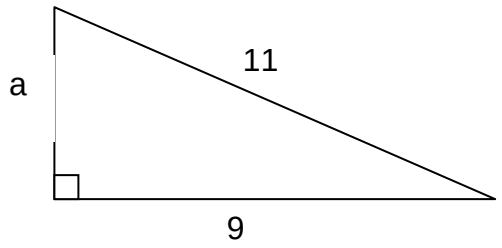
3. Square the given numbers.

4. Add or Subtract?

5. Find the square root.

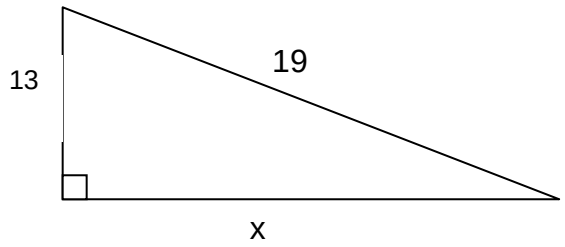
6. Round.

Example: Find **a** to three decimal places.



1. Write the formula.
2. Substitute the given sides.
3. Square the given numbers.
4. Add or Subtract?
5. Find the square root.
6. Round.

Example: Find **x** to three decimal places.

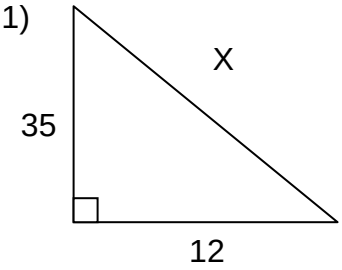


1. Write the formula.
2. Substitute the given sides.
3. Square the given numbers.
4. Add or Subtract?
5. Find the square root.
6. Round.

Homework: See next page!

Find the length of the missing side in each right triangle using $a^2 + b^2 = c^2$.

1)



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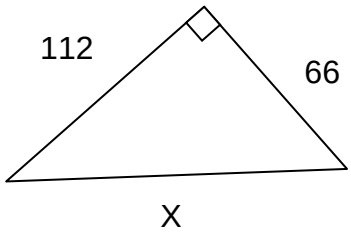
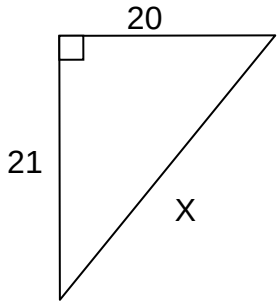
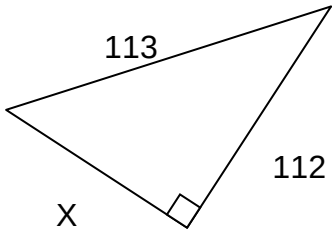


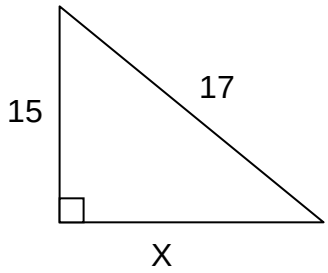
3)



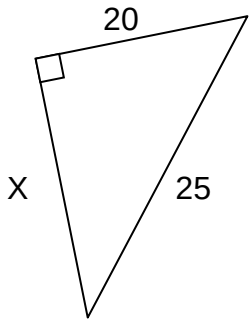
4)



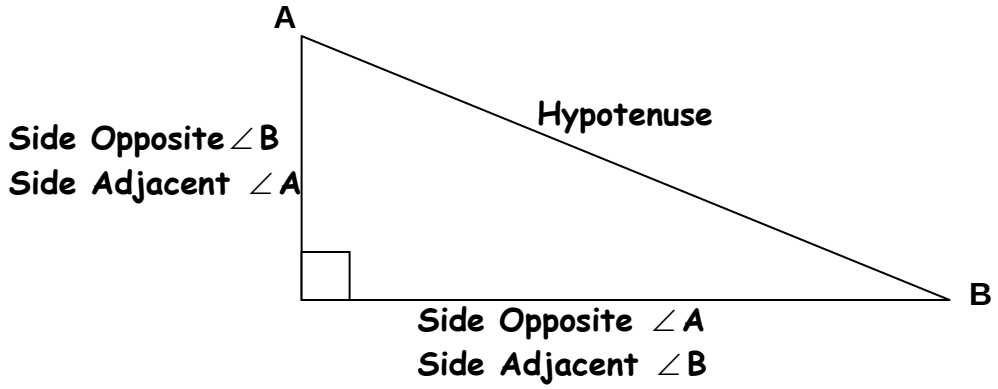
5)



6)



Outcome:10I.M.4 – *Trigonometric Ratios*

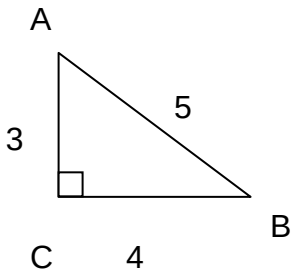


Sine (Sin), Cosine (Cos) and Tangent (Tan)

$\sin\angle A = \frac{\text{Opposite } A}{\text{Hypotenuse}}$ $\cos\angle A = \frac{\text{Adjacent } A}{\text{Hypotenuse}}$ $\tan\angle A = \frac{\text{Opposite } A}{\text{Adjacent } A}$

We can remember this by the word **SOHCAHTOA**

Ex.



$\sin\angle A =$ _____ $\sin\angle B =$ _____

$\cos\angle A =$ _____ $\cos\angle B =$ _____

$\tan\angle A =$ _____ $\tan\angle B =$ _____

Finding Sin, Cos and Tan using a calculator.

Make sure your calculator is in degrees (DEG or D)

When you are given the angle put the value into your calculator and press sin, cos, or tan.

Ex. $\sin 30^0 =$ _____ $\sin 90^0 =$ _____

When you are finding the angle, put in the value you are given and press 2^{nd} then sin, cos, or tan.

Ex. Sinx = 0.3126 x = _____

Cosx = 0.1126 x = _____

Finding Missing Sides and Angles Using Sin, Cos, and Tan

Steps for Finding Missing Sides:

1) What sides is given in relation to the given angle? (i.e. hypotenuse, adjacent, opposite)

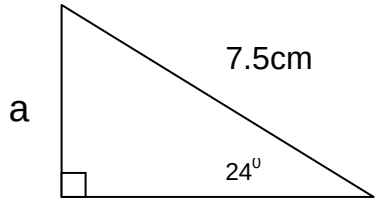
2) What side are you trying to find in relation to the given angle? (i.e. hypotenuse, adjacent, opposite)

3) What trig function (sin, cos, tan) uses those sides? Use SOHCAHTOA

4) Set up and solve the equation.

Example:

Find a.



Follow the 4 steps:

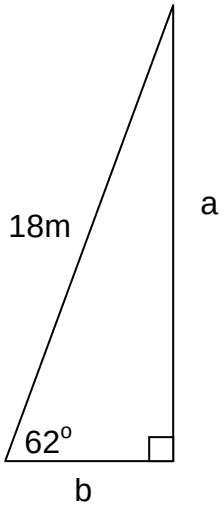
1)

2)

3)

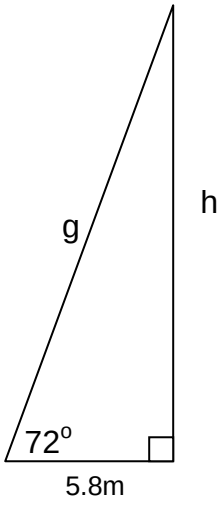
4)

Example: Find a.



Find b.

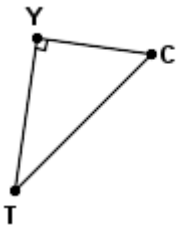
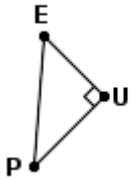
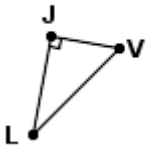
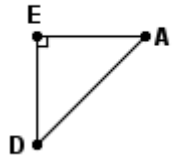
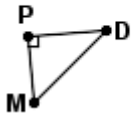
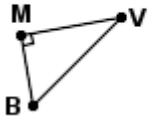
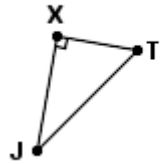
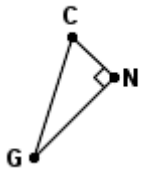
Example: Find g.



Find h.

Homework:

Find the value of x and/or y.

1.  ∠N = 33.06° c = *x* n = 5.4	2.  ∠C = 52.67° c = 5.9 t = *x*	3.  ∠E = 49.42° e = 12.25 u = *x* p= *y*
4.  ∠V = 55.05° l = 5.8 v = *x*	5.  ∠A = 44.99° d = 13.5 e = *x* a = *y*	6.  ∠M = 50.39° p = *x* d = 0.51
7.  ∠B = 54.16° m = *x* v = 2.6	8.  ∠T = 55.13° t = *x* x = 6.46	9.  ∠C = 63.18° n = *x* g = 4.8

Steps for Finding Missing Angles

1) Which sides are given in relation to the angle you want to find.

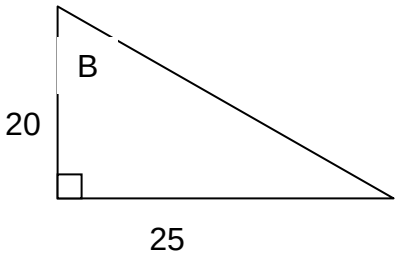
2) What trig function (sin, cos, tan) uses those sides? Use SOHCAHTOA

3) Set up the equation.

4) Divide the fraction and use 2^{nd} sin, cos or tan to find the angle.

Example 1:

Find the angle B.



Solution:

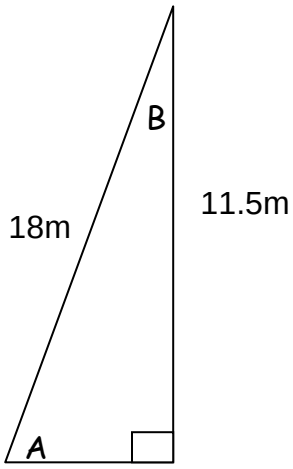
1)

2)

3)

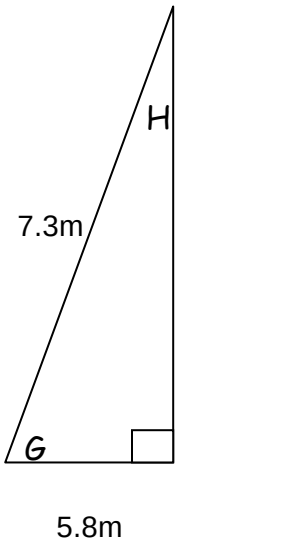
4)

Example 2: Find *A*.



Find B (remember: all angles in a triangle add to 180^0).

Example 3: Find *G*.

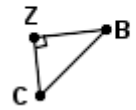


Find H.

Homework:

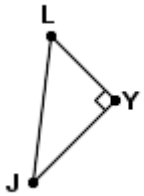
Find the missing angle(s):

1.



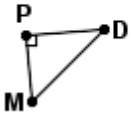
$\overline{BZ}$ = 4
$\overline{CB}$ = 5.12
B =

2.



$\overline{JY}$ = 8.3
$\overline{YL}$ = 6.4
J =

3.

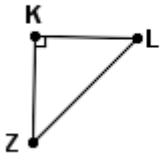


$\overline{DP}$ = 4.33
$\overline{MD}$ = 5.68
M =

4.

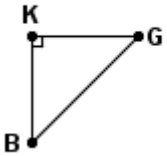
$\overline{MA}$ = 1.72
$\overline{PM}$ = 1.85
P =
M =

5.



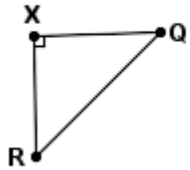
$\overline{ZL}$ = 7.36
$\overline{ZK}$ = 5.3
L =
Z =

6.



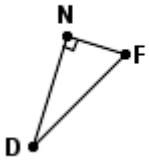
$\overline{BK}$ = 17.7
$\overline{GK}$ = 17.7
B =
G =

7.



$\overline{QX}$ = 4.5
$\overline{RQ}$ = 6.22
Q =
R =

8.



$\overline{DN}$ = 4.5
$\overline{FN}$ = 2.3
F =
D =

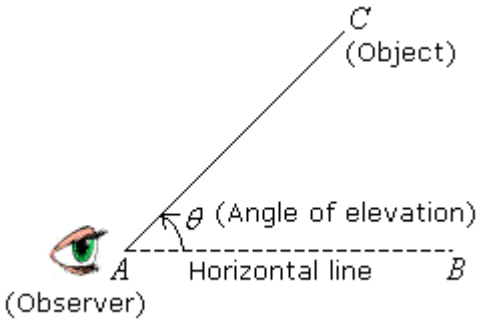
9.

$\overline{HR}$ = 3.42
$\overline{HJ}$ = 3.2
R =
H =

Outcome:10I.M.4 Trigonometry: Angles of Elevation and Depression

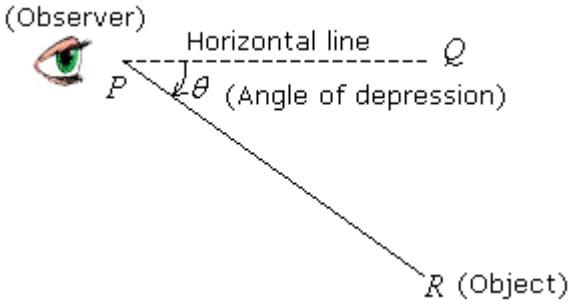
The **angle of elevation** is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

In the diagram below, AB is the horizontal line. q is the angle of elevation from the observer at A to the object at C.



The **angle of depression** is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

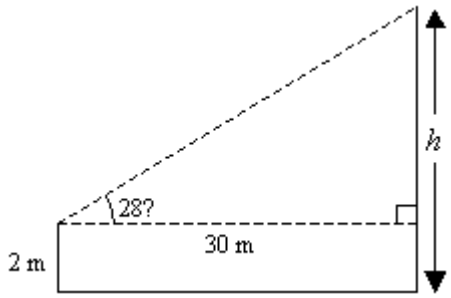
In the diagram below, PQ is the horizontal line. q is the angle of depression from the observer at P to the object at R.



Note: For any given problem, the angle of elevation has the same value as the angle of depression.

Example:

A man who is 2 m tall stands on horizontal ground 30 m from a tree. The angle of elevation of the top of the tree from his eyes is 28°. Determine the height of the tree.



Example:

Kulsoom was standing on the top of a coastal cliff and looking at Laura who was in a boat below. The angle of depression to Laura from Kulsoom was 30°.
The cliff was measured as being 100 meters high.

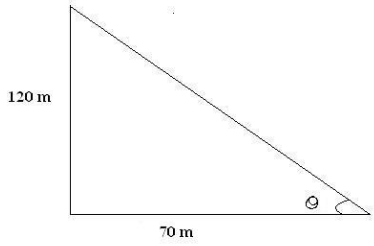
(i) What was the angle of elevation of Kulsoom from Laura?

(ii) Draw a diagram of the above situation and clearly label the angles of elevation and depression.

(iii) How far was Laura from the base of the cliff?

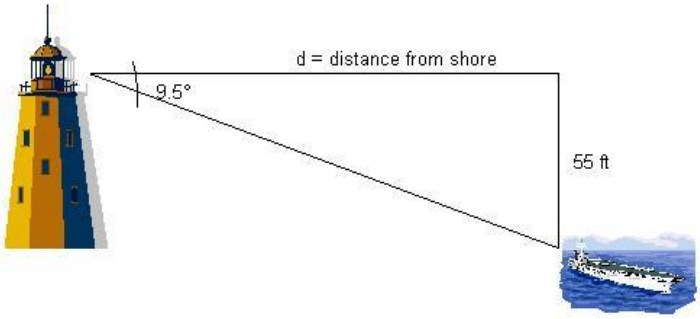
Homework:

1.



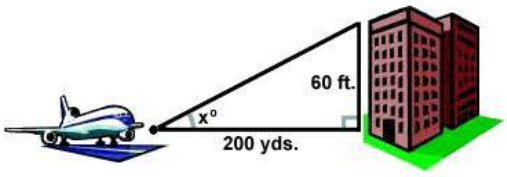
A person is standing 70 m away from a tower. The person is looking at the top of the tower which has the height of 120 m. Find the angle of the elevation.

2.



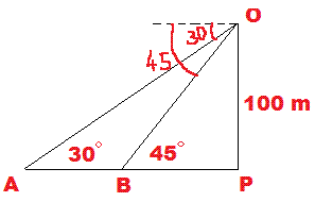
An observer in a 55-foot tall lighthouse spots a ship in distress at an angle of depression of 9.5^0. How far is the ship from shore?

3.



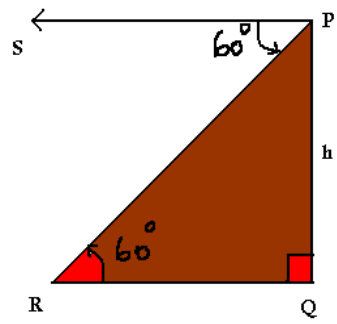
An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant.

4.



As observed from the top of a light house, 100 m above the sea level, the angle of depression of a ship, sailing directly towards it, changes from 30 to 45. Determine the distance travelled by the ship during the period of observation.

5.



The angle of depression of a stone on the ground from the top of a building is 60. If the stone is at a distance of 50 metres away from the building, find the height of the building.

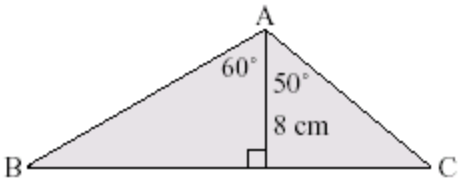
6. John wants to measure the <u>height</u> of a tree. He walks exactly 100 feet from the <u>base</u> of the tree and looks up. The <u>angle</u> from the ground to the top of the tree is 33° . How tall is the tree?

7. A building is 50 feet high. At a distance away from the building, an observer notices that the <u>angle of elevation</u> to the top of the building is 41°. How far is the observer from the <u>base</u> of the building?

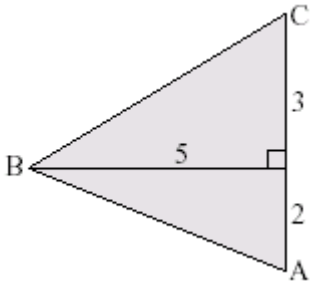
Outcome:10I.M.4 Trigonometry: Solving Problems Using More Than One Right Triangle

For this exercise set, express all lengths to two decimal places and all angles to the degree. Diagrams are not drawn to scale.

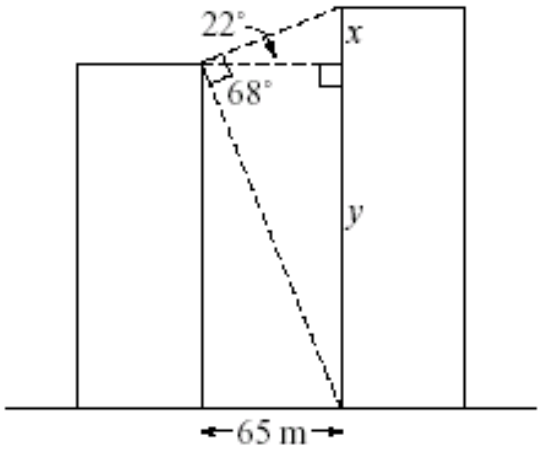
For Example: Calculate the length of BC.



For Example: Calculate the measure of $\angle ABC$.



For Example: Two office towers are 65 m apart. The angle of depression from the top of the shorter tower to the base of the taller tower is 68°. The angle of elevation from the same point to the top of the taller tower is 22°.

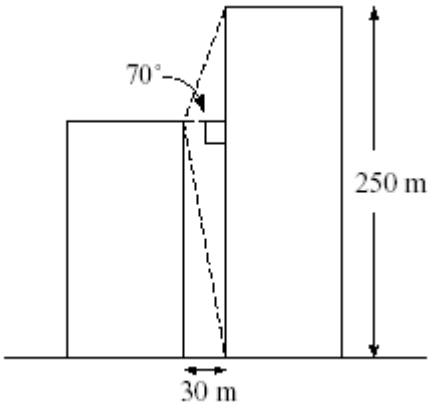


a) Calculate x and y.

b) What is the height of the taller tower?

Homework:

1. Two office towers are 30 m apart. From the top of the shorter tower, the angle of
elevation to the top of the other tower, which is 250 m high, is 70°.

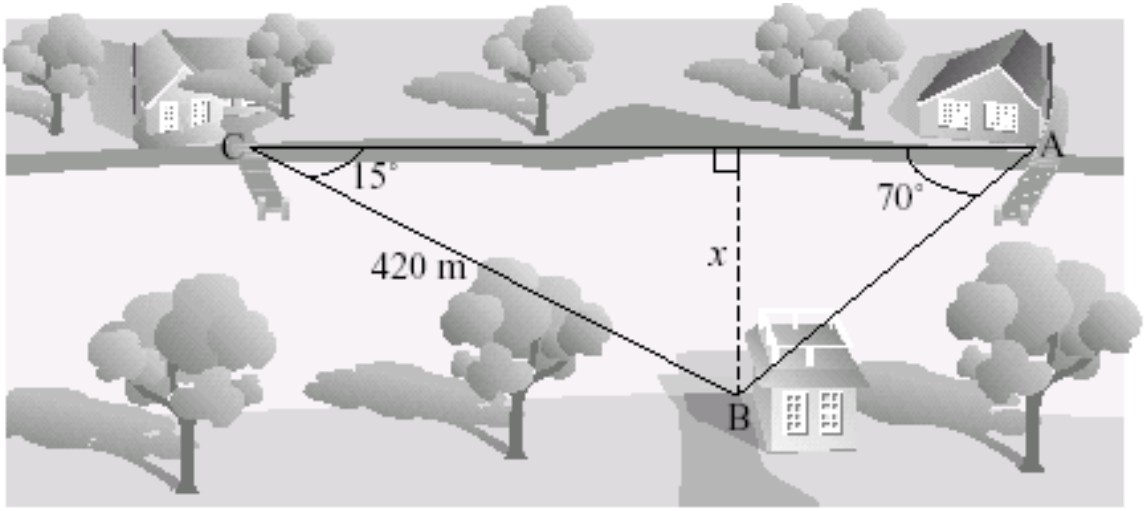


a) How much taller is the taller tower?

b) Determine the height of the shorter tower.

c) Determine the angle of depression to the base of the taller tower from the top of the shorter tower.

2. Two cabins, A and C are located a distance apart on the bank of a river. On the Other side of the river from the two cabins is a boathouse, B. It is 420 m from cabin C to the boathouse and the angle at cabin C between the boathouse and cabin A is 15°. From cabin A, the angle between cabin C and the boathouse is 70°.

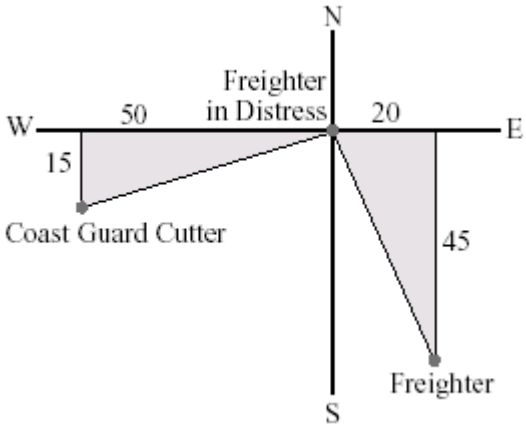


a) Determine x.

b) How far is cabin A from the boathouse?

c) How far apart are the cabins?

3. An ocean freighter sends a distress signal. A coast guard cutter that is 50 nautical miles west and 15 nautical miles south of the freighter hears the distress call. A second freighter that is 20 nautical miles east and 45 nautical miles south of the freighter in distress also hears the call.



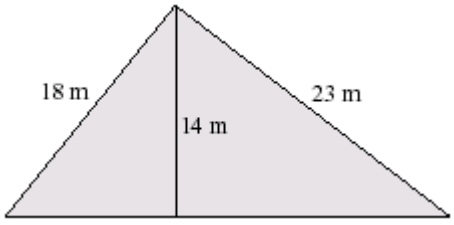
a) Which vessel is closer to the ship in distress and by how much?

b) At what angle from north would this vessel have to head to reach the ocean freighter?

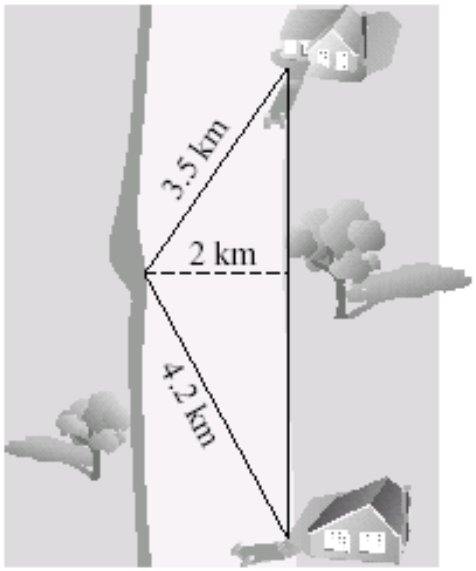
4. A non-standard roof on a house has one side 18 m long and the other side is 23 m long. The peak is 14 m high.

a) Determine the measure of the angle formed at the peak.

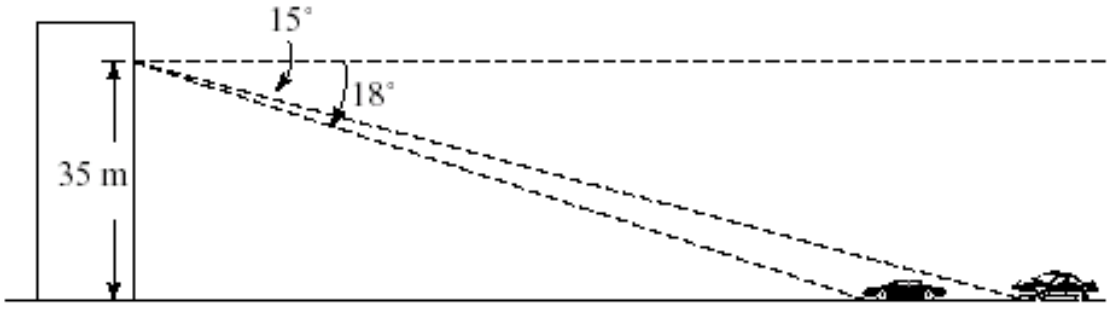
b) What is the span of the roof?



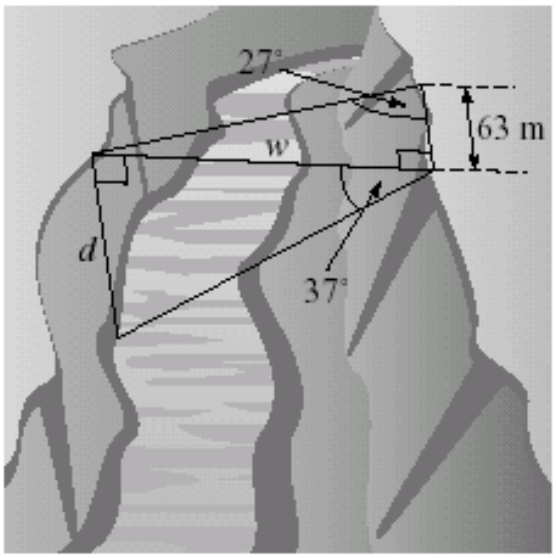
5. From a point on the west bank of a river 2 km wide, two speedboats leave for their respective cabins on the east side of the river. The distance to the closer cabin is 3.5 km and the distance to the further cabin is 4.2 km. What is the measure of the angle between the boats' paths?



6. Olivia looks out the window of her apartment building and sees a Corvette down the street at an angle of depression of 18°. A little farther down the street, she sees a police car at an angle of depression of 15°. Her apartment window is 35 m above street level. How far apart are the Corvette and the police car?



7. For the gorge in the diagram below, calculate:



a) the width (*w*)

b) the depth (*d*)

8. From the top of a 100 m tower, a fire ranger spots two fires. One fire is due east of the tower at an angle of depression of 16°. The other is due west and has an angle of depression of 23°. Calculate the distance between the fires.