Unit: Solving Quadratic Equations

Name ________________________ Dates Taught ________________

Outcome	
11P.R.1.	Factor polynomial expressions of the of the form ○ $ax^2 - bx + c = 0, a \neq 0$ ○ $a^2x^2 - b^2y^2 - c = 0, a \neq 0\ b \neq 0$ ○ $a(f(x))^2 - b(f(x))x + c = 0, a \neq 0$ ○ $a^2(f(x))^2 - b^2(g(y))^2 - c = 0, a \neq 0\ b \neq 0$ where a, b and c are rational numbers. Achievement Indicators • Factor a polynomial expression that requires the identification of common factors. • Determine whether a binomial is a factor for a polynomial expression, and explain why or why not. • Factor a polynomial expression of the form ○ $ax^2 - bx - c = 0, a \neq 0$ ○ $a^2x^2 - b^2y^2 - c = 0, a \neq 0\ b \neq 0$ • Factor a polynomial expression that has a quadratic pattern, including ○ $a(f(x))^2 - b(f(x))x + c = 0, a \neq 0$ ○ $a^2(f(x))^2 - b^2(g(y))^2 - c = 0, a \neq 0\ b \neq 0$
11P.R.5	Solve problems that involve quadratic equations. Achievement Indicators • Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the x-intercepts of the graph of the quadratic function. • Derive the quadratic formula, using deductive reasoning. • Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as ○ determining square roots ○ factoring ○ completing the square ○ applying the quadratic formula ○ graphing its corresponding function • Select a method for solving a quadratic equation, justify the choice, and verify the solution. • Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function. • Identify and correct errors in a solution to a quadratic equation. • solve a problem by determining or analyzing a quadratic equation.

Factoring Polynomial Expressions - Review

Monomial by Monomial:

$(4x)(7x^2) =$ ______

$(-6m^2n^3)(-7mn^2) =$ ______

Monomial by Binomial: Distributive Property

$3x(x - 2) =$ ______

$5y^2(x^2 - y) =$ ______

Monomial by Trinomial:

$abc(3a + 4b - 2c) =$ ______

$(2y^2 + 3y - 1)(4y) =$ ______

Binomial by Binomial (F. O. I. L.):

$(2x - y)(3x + y) =$ ______

$=$ ______

$(x + 6)(x + 8) =$ ______

$=$ ______

$(x - 2y)(x + 2y) =$ ______

$=$ ______

$-3(x - 3y)(2x + 5y) =$ ______________________________

$=$ ______________________________

$=$ ______________________________

Binomial Squared: (F. O. I. L.)

$(x + y)^2 =$ ______________________________

$=$ ______________________________

$(x + 5)^2 =$ ______________________________

$=$ ______________________________

$(2x - y)^2 =$ ______________________________

$=$ ______________________________

$-4(3x + y)^2 =$ ______________________________

$=$ ______________________________

$=$ ______________________________

Prerequisite Skills: Simplify the following

a) $7x^2 - 3x + x^2 - x$

b) $(4x - 3)(x + 7)$

b) $(2x - 5)^2$

d) $(x - 1)^2 - (2x + 3)(x - 4)$

e) $3(2x - 7) - 4(x - 1)$

f) $5x(3x - 2)$

g) $(4x - 3)(2x + 5)$

h) $(5x - 4)^2$

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Binomial Products

1. $(a + 3)(a + 2) =$

2. $(x - 1)(x - 2) =$

3. $(2 + k)(3 + k) =$

4. $(c - 5)(c - 3) =$

5. $(y - 4)(y + 6) =$

6. $(y + 5)(t - 1) =$

7. $(3 - b)(4 + b) =$

8. $(6v + 3)(v + 2) =$

9. $(5 + 3x)(2 + x) =$

10. $(y - 5)(2y - 2) =$

11. $(m + 3)(3m - 2) =$

12. $(2a + 3)(3a + 2) =$

13. $(4 + 3p)(3 - 4p) =$ ______

14. $2(a + 3)(a + 2) =$ ______

15. $-3(y - 7)(y + 5) =$ ______

16. $(a - 3)(a + 3) =$ ______

17. $(5a + 3)(5a - 3) =$ ______

18. $(a^2 - 6)(a^2 + 6) =$ ______

19. $(a + 4)(a + 4) =$ ______

20. $(x - 3)^2 =$ ______

21. $(2c + 5)^2 =$ ______

22. $(4a - 3b)^2 =$ ______

23. $(y + 3)(y^2 - 7y + 5) =$ ______

24. $(a^2 - 3a + 11)(a - 3) =$ ______

25. $(2x - 3)(2x^2 - 3xy + y^2) =$ ______

Exercise 1: Factoring Review (Common and Simple Trinomial)

Factoring is the reverse process of ____________________. The better you are at multiplying, the better you will be at factoring.

Multiplication →	Factoring →
$5x(x - 2y) = 5x^2 - 10xy$	$9x^2 - 15x = 3x(3x - 5)$
$(x - 3)(x + 5) = x^2 + 2x - 15$	$x^2 + 8x + 15 = (x + 3)(x + 5)$

Common Factoring:

- When factoring, always begin by looking for any numbers or variables that is (are) common to (appears in) every term. It could be a number, a variable or both.
- Place this common factor in front of the parentheses, with the remaining polynomial inside the parentheses.
- Once this is done, the same number of terms as in the original question should be inside the brackets.

Examples: $8x - 32y$ = ________(__________)
Common factor   Remaining factor

$b - b^2r^3c$ = ________(__________)
Common factor   Remaining factor

$3x^3 - 6x^2y + 9xy^2$ = _______(________________)
Common factor   Remaining factor

- Factoring can always be quickly and easily checked by _______________ the ________________ together to see if the result is the same as the original polynomial.

Trinomial Factoring:

- When factoring a trinomial, you need to ask yourself: " *Where did it come from?*" Trinomials are most commonly the result of multiplying two binomials using ____________.
- For now, we will consider only trinomials where the lead coefficient in front of the "x^2" is one. If it is not 1, try to make it one by factoring out a common value amongst all terms.

Example:

$x^2 + 5x + 6$

First Terms → Sum / Difference / Outside / Inside → Last Terms

(__________)(__________)

① Once you have factored out any common terms/variables, the first terms of each binomial will simply be the square root of the first term of the trinomial.

② To determine the last terms of each binomial, we need to first identify the factors of the last term of the trinomial. In this case, the factors of 6 are: ___________ ; and __________

③ Next, determine which pair of factors either adds up to or subtracts to get the _______term. (In this case:_____). These factors become the last terms of the binomial.

④ Therefore, $x^2 + 5x + 6$ factors to (________)(_______)

Check your factoring by multiplying using FOIL!

- There are <u>4</u> *types* of simple trinomials:

1. $x^2 + 8x + 12$ Sign in the ________ is _______________ in both parentheses. ⇒ $(x + 2)(x + 6)$
2. $x^2 - 8x + 16$ Sign in the ________ is _______________ in both parentheses. ⇒ $(x - 4)(x - 4)$
3. $x^2 - 5x - 24$ One middle sign is______________, one is___________. ⇒ $(x + 3)(x - 8)$
4. $x^2 + 3x - 54$ One middle sign is______________, one is___________. ⇒ $(x + 9)(x - 6)$

Examples: Factor the following trinomials:

1. $x^2 + 9x + 18$ ① **Common** terms? Y/N The first term: ____ ② **Factors** of ____: ③ Which of these **pairs** **add/subtract** to _____

④ ⇒ (________)(________)

More Examples:

2. $x^2 - 2x - 15$ =____ ④ ⇒ (________)(________)

3. $x^2 - 6x + 5$ =____ ④ ⇒ (________)(________)

4. $a^2 - 3a + 2$ =____ ④ ⇒ (________)(________)

5. $a^2 + 10a + 24$ =____ ④ ⇒ (________)(________)

6. $x^2 - 4x - 60$ =____ ④ ⇒ (________)(________)

7. $x^2 + 8x$ =____ ④ ⇒ (_____)(_______)

8. $b^2 + b - 12$ =____ ④ ⇒ (________)(________)

9. $2x^2 + 8x + 6$ =____

=____④ ⇒__(___)(________)

10. $xy^2 + xy - 13y$ =____

=____ ④ ⇒ (___)(______________)

11. $x^2 - 6x + 9$ =____ ④ ⇒ (_____)(_____) = (_____)

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Factoring Using F.O.I.L.

1. $x^2 + 10x + 24 =$ ______________________________

2. $p^2 + 8p + 16 =$ ______________________________

3. $c^2 - 17x + 72 =$ ______________________________

4. $a^2 – 7a + 6 =$ ______________________________

5. $x^2 - 14x + 49 =$ ______________________________

6. $d^2 – 4d - 45 =$ ______________________________

7. $m^2 – 15m + 50 =$ ______________________________

8. $k^4 + 11k^2 + 30 =$ ______________________________

9. $x^2 + 5x + 5 =$ ______________________________

10. $a^2 – 5a + 4 =$ ______________________________

11. $y^2 + 12y + 35 =$ ______________________________

12. $z^2 – 12z + 35 =$ ______________________________

13. $v^2 + 2v - 35 =$ ______________________________

14. $d^2 – 2v - 35 =$ ______________________________

15. $2x^2 + 12x + 10 =$ ______________________________

16. $4y + 10 =$ ______________________________

17. $6m^2 + 9m =$ ______________________________

18. $4t^3 – 6t^2 =$ ______________________________

19. $3p^3q^2r^2 – 4p^2q^2r =$ ______________________________

20. $5a^2 + 10ab – 15b^2 =$ ______________________________

<u>Exercise 1</u>: Factoring $ax^2 + bx + c$

- Recall: Factoring is the inverse of multiplication.

Example: Factor $15x^2 + 48x + 36$

Product-Sum-Factor **(PSF) Method**	
	① ALWAYS begin factoring by checking for common factors within each term.
	② From the remaining trinomial, calculate the product of the first and last coefficients.
	③ List all of the factors of this product. (List them as pairs)
	④ ④ From these *pairs* of factors, identify which pair, when <u>*added*</u> together, result in the middle term of the trinomial.
	⑤ Re-write the trinomial with the original first and last terms. In between these two terms, insert two new terms using the pair of factors from step ④. When inserting these two new terms, think about which two should go together. That is, which two coefficients have a common factor?
	⑥ Now factor the first two and the last two terms. (Notice how a common factor emerges.)
	⑦ Now factor again.
	⑧ **Check** your answer by expanding (multiplying using FOIL)

① Any common terms?
② Product of 1st & last coefficients?
③ Product factors (pairs)?
④ Which pair = Sum of middle term?
⑤ Re-write trinomial ⇒ 4 monomials.
⑥ Factor 1st two terms & last two.
⑦ Factor again.

In summary, to factor a trinomial of the form $ax^2 + bx + c$, look for **two integers** with a **sum of b** and a **product of ac**.

Examples: Factor the following fully, *if possible*:

1. $3x^2 + 17x + 10 =$

2. $3y^2 - 10y + 8 =$

3. $8a^2 + 18a - 5 =$

4. $2c^2 + 2c - 3 =$

5. $5x^2 - 20xy + 20y^2 =$

6. $6b^4 + 7b^2 - 10 =$

7. $2d^3 + 7d^2 - 30d =$

8. $12 + 18e + 8e^2 =$

9. $9f^2 - 15f - 4 =$

10. $10g^2 - 3gh - h^2 =$

11. $6k^2 + 14km - 12m^2 =$

12. $24x^2z + 38xyz - 36zy^2 =$

<u>Exercise 1</u>: Factoring Difference of Squares (& *Perfect* Squares)

A) Difference of Squares: $ax^2 - by^2$

- A difference of squares has 3 main features:
  1. The first term is a *perfect square*.
  2. The second term is a *perfect* square.
  3. They are separated by a *minus* sign.

Example:
$x^2 - 16y^2$

- The middle term is missing because it is **0**.

Eg. $x^2 - 0xy - 16y^2$

- Factoring a perfect square *binomial* results in 2 almost identical factors, that differ only in the middle negative and positive signs. **Note: You can't factor if both middle terms are "+".**

- **To factor a difference of squares:**
  ① Remember to always begin factoring by looking for a **common** factor.
  ② The first term of the factors comes from the square root of the first term.
  ③ The second term of the factors comes from the square root of the second term.
  ④ Place a <u>negative</u> sign in one parentheses and a <u>positive</u> in the other.
  ⑤ **Check** the result by using FOIL.

Example from above: $x^2 - 16y^2$ = (________)(________)

Examples:

1. $x^2 - 9$ = (__________)(__________)

2. $225b^2 - a^2$ = (____________)(_____________)

3. $49 + x^2$ = (_________)(__________)

4. $-y^2 + 36$ = (________)(__________)

5. $3x^3 - 48x$ =

=

6. $x^4 - 16$ =

B) Perfect Square *Trinomials*: $ax^2 \pm bxy + cy^2$

Example:
$x^2 - 8xy + 16y^2$

- A *perfect* square trinomial has *4* main features:
  1. The first term is a *perfect square*.
  2. The third term is a *perfect square*.
  3. The *sign* of the *3rd* term is always *positive*.
  4. The *middle* term is always *twice* the sum/difference of the outside/inside product of the binomials.

- Factoring a perfect square *trinomial* results in two **identical binomials**.

- **To factor a perfect square trinomial:**

① Remember to ALWAYS begin factoring by looking for a **common factor**.
② The first terms of the factors come from taking the square root of the first term of the trinomial.
③ The 2nd terms of the factors comes from taking the square roots of the 3rd term of the trinomial.
④ The middle signs of the factors will always be either positive or negative, depending on the sign of the middle term of the trinomial.
Remember: The last term of the trinomial is +. ($+ \times + = +$ or $- \times - = +$)
⑤ **Check** the result by using FOIL.

Example: Factor: $x^2 - 8xy + 16y^2$ = (__________)(__________) =

Check:

Examples: Factor the following trinomials (1-5) fully, if possible.

1. $49 + 14x + x^2$ = (__________)(__________)

2. $5b^3 - 40b^2 + 80b$ = _____(____________________)

= _____(_________)(_________)

3. $4p^2 + 20pq + 25q^2$ =

4. $16y^2 + 24y - 9$ =

5. $121x^2 - 22x + 1$ =

6. The *volume* of a *rectangular* prism is represented by $2x^3 - 24x^2 + 72x$. What are possible *dimensions* of the prism?

=______(____________________)

= ____

Practice - Factoring ($ax^2 + bx + c$ and perfect squares)

Name: ____________________________ Date: ________________________

Factor the following polynomials completely.

1. $5a^2 + 15a - 20$

2. $2b^2 + 2b - 112$

3. $9c^2 + 54c + 45$

4. $3d^2 + 15d - 108$

5. $7e^2 - 84e - 196$

6. $g^2 + 5gh + 6h^2$

7. $x^2y^2 + 5xy + 6$

8. $5km^2 - 40km + 35k$

9. $f^4 + 9f^2 + 14$

10. $2n^4 + 16n^2 + 30$

11. $6x^3y - 7x^2y^2 - 3xy^3$

12. $4s^2 - 21st - 18t^2$

13. $p^2 - 64$

14. $9r^2 - 81$

15. $16 - 49u^2$

16. $25v^2 - 169w^2$

17. $1 - 16x^4$

18. $y^4 - 16$

19. $63a^2b - 28b$

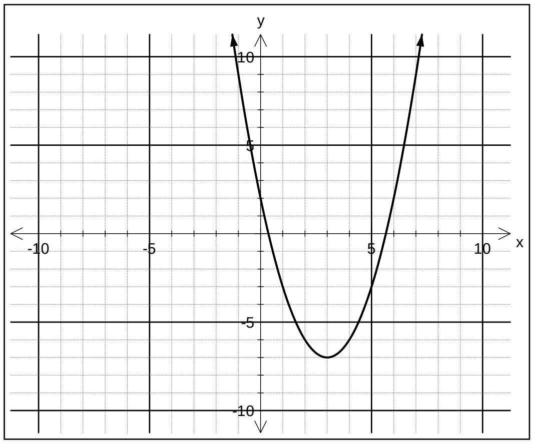
20. $81x^2 - (3x + y)^2$

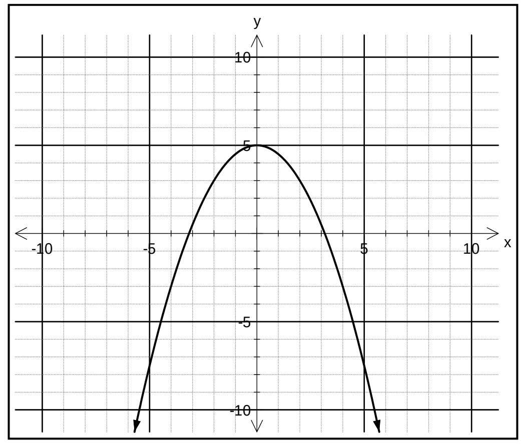
21. $(5m - 2)^2 - (3m - 4)^2$

22. $(3y + 8z)^2 - (3y - 8z)^2$

<u>Exercise 2</u>: Solving Quadratic Equations by Factoring

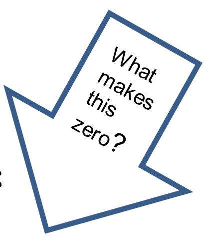
A ____________________equation is any __________________that can be written in the form:____________________, where a, b, and c are constants and a ≠ 0. There graph is in the shape of a ______________________.



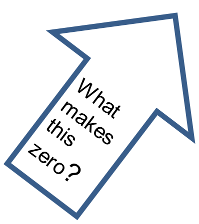


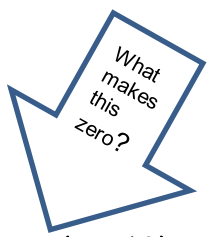
Quadratic Equations cannot be__________________by isolating the __________________ as in Linear Equations. One of the strategies you can use is to __________________. After factoring, one or more of the factors could be __________________ which would make the equation zero. These factors are the _______________ or x-__________________ of the parabola.

For example:

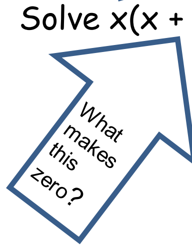


Solve $(x - 9)(x + 3) = 0$





Solve $x(x + 12) = 0$



For example:

Solve $x^2 + 3x - 28 = 0$

Solve $x^2 - 7x = 0$

Solve $x^2 + x = 6$

Solve $4x^2 - 9x = 28$

The answers to a Quadratic Equation are called ___________________.

Further Examples:

Solve $3x^2 + 4x - 4 = 0$

Solve $2x^2 + 2x - 24 = 0$

Solve $3x^2 - \frac{1}{12} = 0$

Solve $0.5x^2 - 0.02 = 0$

Solving Radical Equations

A radical equation is an equation in which a variable appears under a radical sign

Procedure:

Step 1: Isolate a radical on one side of the equation

Step 2: Square both sides/Simplify

Step 3: If any radicals remain, repeat steps 1 and 2

Step 4: Simplify and solve equation

Step 5: Check by substitution in original equation

Note: There may be **extraneous roots** as squaring can equate things that are not equal.

For Example:

Solve: $\sqrt{3x-8} + 1 = 3$ Solution:

Raising both sides of an equation to the *n*th power may introduce extraneous or false solutions. So when you use the procedure, it is critical that you check each solution in the original equation.

For Example:

Solve: $1 + \sqrt{4x+8} = x$ Solution:

Check each solution. Note that this equation has only one solution, ____. The number ____ is called an extraneous root as it does not satisfy the equation.

For Example:

Solve: $\sqrt{\sqrt{x^2+6x}} = 2$ Solution:

For Example:

Solve: $\sqrt{2x+1}+\sqrt{x}=5$ Solution:

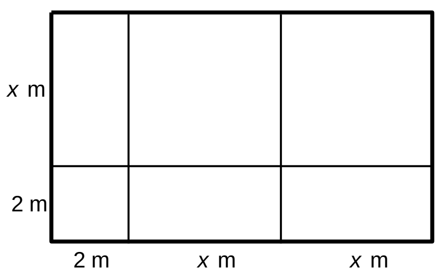
For Example:

Solve: $\sqrt{2x+5}-\sqrt{x-2}=3$ Solution:

Problems

For Example:

The total area of the large rectangle below is 24 m^2. Determine the value of x.



Solution:

For Example:

The perimeter, P, of a rectangular concrete slab is 46 m and its area, A, is 90 m^2.
Use the formula $P = 2l + 2w$. Determine the dimensions of the slab. Show your work.

Solution:

For Example:

A student wrote the solution below to solve this equation: $(4x+1)^2 = (2x-3)^2$

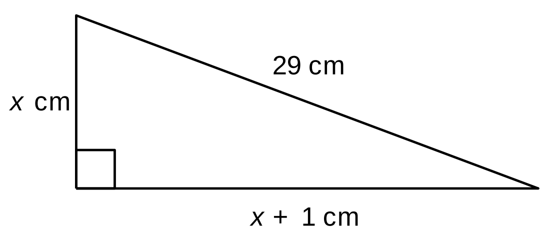
$$\begin{aligned}(4x+1)^2 &= (2x-3)^2 \\ 4x+1 &= 2x-3 \\ 4x+1-2x+3 &= 0 \\ 2x+4 &= 0 \\ 2(x+2) &= 0 \\ x &= -2\end{aligned}$$

Identify the error, then write the correct solution.

Solution:

For Example:

Determine the lengths of the legs in this right triangle. Explain your strategy.



Solution:

Exercise 5: Using Square Roots to Solve Quadratic Equations

Some quadratic equations end up with square roots as a solution. When you solve an equation by taking the square root of both sides you get 2 solutions: the positive and negative roots.

For Example:

$x^2 + 2 = 11$

Solution:

For Example:

$3x^2 - 2 = 13$

Solution:

Note: Leave answers as radicals when roots are not perfect. That way your answer is exact.

For Example:

$(x - 1)^2 - 7 = 24$

Solution:

For Example:

$(3x - 5)^2 - 2 = 12$

Solution:

For Example:

$x^2 + 10 = 1$

Solution:

For Example:

$-3x^2 - 12 = 4$

Solution:

These 2 examples have solutions that are not real numbers. Since, x^2 cannot be negative, we say the equation has ***no real roots***. (Only imaginary roots!!!!)

Completing the square:

When Quadratic Equations can't be solved by factoring, we need to be resourceful.

Using the Completing the Square Method to convert $f(x) = ax^2 + bx + c$ into $f(x) = a(x - p)^2 + q$

To use this method, we need to create a Perfect Square trinomial, one that can be factored into two identical binomials.

Eg. $x^2 + 6x + 9 = (\qquad)(\qquad)$

For Example:

What must be added to $x^2 + 4x$ to make it a Perfect Square?

Solution:

For Example:

What must be added to $x^2 + 6x$ to make it a Perfect Square?

Solution:

For Example:

What must be added to $x^2 - 16x$ to make it a Perfect Square?

Solution:

For Example:

What must be added to $x^2 + 3x$ to make it a Perfect Square?

Solution:

Hint:____

Steps to Completing the Square(and solving):

Complete the square for $0 = x^2 + 4x + 7$ and solve.

Solution:

1. Build a perfect square trinomial from the first two terms.

2. Compensate. The number added to create the perfect square must be subtracted to keep the value of the function the same.

3. Factor and simplify into $f(x) = a(x - p)^2 + q$ form.

4. From there, solve for x using square roots. (Q.A.S.P.-???)

For Example:

Complete the square of $0 = x^2 - 6x + 2$ and solve. Write the equation in the form $0 = a(x - p)^2 + q$.

Solution:

For Example:

Complete the square of $0 = -x^2 + 8x - 3$ and solve. Write the equation in the form $0 = a(x - p)^2 + q$.

Solution:

For Example:

Complete the square of $0 = 2x^2 + 4x - 9$ and solve. Write the equation in the form $0 = a(x - p)^2 + q$.

Solution:

For Example:

Complete the square of $0 = \frac{1}{2}x^2 - 4x + \frac{1}{2}$ and solve .

Solution:

For Example:

A student wrote the solution below to solve this quadratic equation: $2x^2 - 12x - 13 = 0$

$$2x^2 - 12x - 13 = 0$$
$$2x^2 - 12x = 13$$
$$2(x^2 - 6x) = 13$$
$$2(x^2 - 6x + 9) = 13 + 9$$
$$2(x - 3)^2 = 22$$
$$(x - 3)^2 = 11$$
$$x - 3 = \pm\sqrt{11}$$
$$x = 3 \pm \sqrt{11}$$

Solution:

The roots are: $x = 3 + \sqrt{11}$ and $x = 3 - \sqrt{11}$

Identify the error, then write the correct solution.

For Example:

Complete the square of 0 = -2x^2 + 3x + 5 and solve.

Solution:

Name:______________________________

Completing the Square – Further Examples

1) $y = x^2 + 12x + 28$

2) $y = 2x^2 - 10x$

3) $y = 24x - 8x^2$

4) $y = -3x^2 - 12x$

5) $y = \frac{1}{2}x^2 - 3x$

6) $y = x^2 + x + \frac{5}{4}$

7) $y = 3x^2 + 6x + 8$

8) $y = -3x^2 + 24x - 37$

9) $y = \frac{1}{2}x^2 - 2x + \frac{1}{2}$

10) $y = 2x^2 - 6x + \frac{13}{2}$

Mental Math:

1. If $x^2 + 14x + k = (x + 7)^2$, then k = ?

2. Factor $x^2 - 5x + 6$.

3. Expand $(x + 2)^2$.

4. Factor $x^2 + 10x + 25$.

5. $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3}) = a$. Find a.

6. For what value of m is $x^2 - 4x + m$ a perfect square.

7. For what value of m is $x^2 - 5x + m$ a perfect square.

8. For what value of m is $x^2 + \frac{1}{2}x + m$ a perfect square.

<u>Exercise 6</u>: Developing and Applying the Quadratic Formula

The Quadratic Formula can be used to solve **any** quadratic equation. When using the quadratic formula, you must set up the equation in the standard form $ax^2 + bx + c = 0$, $a \neq 0$, so that a, b, and c can be correctly identified.

If $ax^2 + bx + c = 0$, where $a \neq 0$, the exact solutions(roots) are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ are the two roots(zeroes) of the equation.

For Example:

What is the coefficient of x in the quadratic equation, $-x + 8x^2 + 6 = 0$?

Solution:

For Example:

Identify the values of a, b, and c in this quadratic equation: $3x^2 + 4x + 8 = 0$

Solution:

For Example:

Solve $x^2 + 3x - 9 = 0$ using the quadratic formula.

Solution:

For Example:

Solve $2x^2 - 6x - 7 = 0$

Solution:

For Example:

Solve this quadratic equation: $-11 - 2x + 3x^2 = 0$. Give the solution to 2 decimal places.

Solution:

For Example:

Consider this quadratic equation: $-x^2 + \frac{2}{3}x - \frac{1}{2} = 0$

a) Rewrite the equation so that it does not contain fractions.
b) Solve the equation. Explain your answer.

Solution:

For Example:

Solve $2x^4 - 5x^2 + 2 = 0$

Solution:

For Example:

The quadratic equation $x^2 + bx + 6 = 0$ has two positive integral roots. What are the possible values of b?

Solution:

For Example:

Marc's rectangular garden measures 7 m by 10 m. He wants to double the area of his garden by adding equal lengths to both dimensions. Determine this length to the nearest tenth of a metre. Show your work.

Solution:

Mental Math

1. Factor: $x^2 - 2x + 1$

2. Factor: $x^2 - 2x - 8$

3. What is the value of *a* in the standard form of the quadratic equation $7x^2 - 5x + 1 = 0$?

4. What is the value of *c* in the quadratic equation $2x^2 = 5x - 8$?

5. What is the value of *b* in the equation $0.5x^2 = 2$?

Exercise 7: Interpreting the Discriminant

When using the Quadratic Formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The expression **$b^2$ - 4ac** is called the discriminant and it enables you to find how many real number solutions a quadratic equation has without solving the equation or graphing.

For Example:

Solve the following quadratics and state how many real number solutions they have. Evaluate the discriminant $b^2 - 4ac$ for all the equations. What do you notice?

Solution:

a) $x^2 - x - 12 = 0$

b) $-x^2 + 2x + 24 = 0$

c) $x^2 - 4 = 0$

d) $x^2 + 8x + 9$

For Example:

Solve the following quadratics and state how many real number solutions they have. Evaluate the discriminant $b^2 - 4ac$ for all the equations. What do you notice?

Solution:

a) $x^2 - x + 2 = 0$

b) $x^2 + 6x + 10 = 0$

c) $-x^2 + x - 4 = 0$

For Example:

Solve the following quadratics and state how many real number solutions they have. Evaluate the discriminant $b^2 - 4ac$ for all the equations. What do you notice?

Solution:

a) $x^2 - 6x + 9 = 0$

d) $x^2 + 4x + 4 = 0$

c) $-x^2 + 10x - 25 = 0$

Note:

1. If $b^2 - 4ac > 0$ and is a non-perfect square, then the equation has 2 real irrational roots.

2. If $b^2 - 4ac > 0$ and is a perfect square, then the equation has 2 real rational roots.

3. If $b^2 - 4ac = 0$, then the equation has 1 real number root.

4. If $b^2 - 4ac < 0$, then the equation has 0 real number roots, but the roots are described as imaginary.

And

1. If $b^2 - 4ac > 0$, then the graph has 2 real zeros.

2. If $b^2 - 4ac = 0$, then the graph has 1 real zero (is tangent to x-axis).

3. If $b^2 - 4ac < 0$, then the graph has no real zeros.

For Example:

Solve for x: $x^2 - 6x + 7 = 0$

Solution:

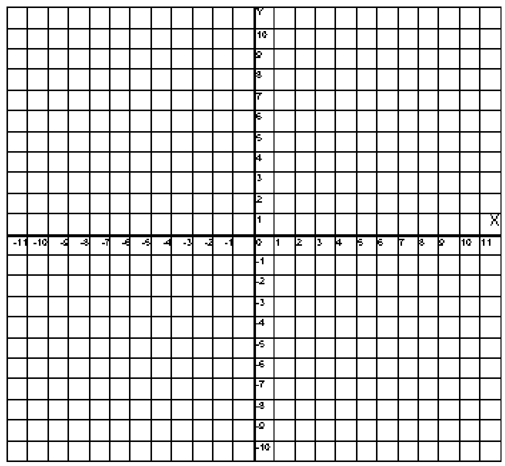
Equation:

Value of discriminant:

Roots of Quadratic:

Description of Roots:

Graph of the quadratic function: $y = x^2 - 6x + 7$



Description of the zeros:

For Example:

Solve for x: $x^2 - 6x + 14 = 0$

Solution:

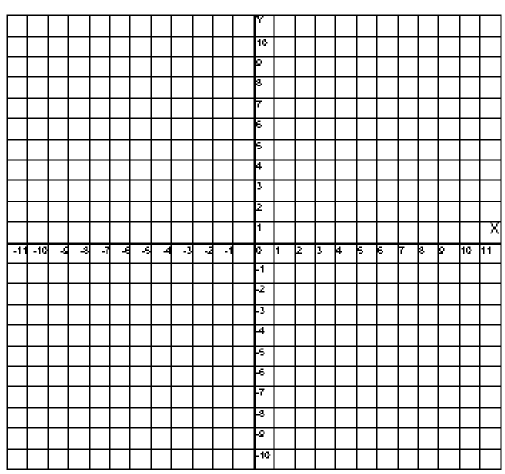
Equation:

Value of discriminant:

Roots of Quadratic:

Description of Roots:

Graph of the quadratic function: $y = x^2 - 6x + 14$



Description of the zeros:

For Example:

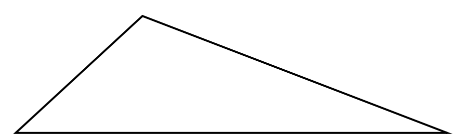
For what values of *m* will $x^2 + 8x - m$ have real and unequal roots?

Solution:

For Example:

In triangle ABC, a = 2, b = 6, $\angle A = 30°$. Instead of using the Sine Law, solve the triangle using the Cosine law. ($a^2 = b^2 + c^2 - 2bcCosA$).

Solution:



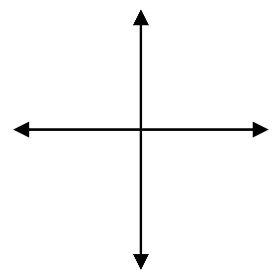
Mental Math

1.If the discriminant of a quadratic equation is negative, how many roots does it have?

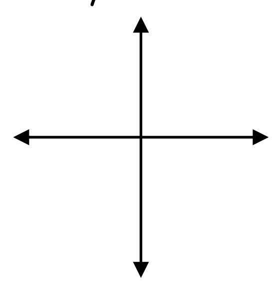
2.If the discriminant of a quadratic equation is positive, how many roots does it have?

3. Calculate the discriminant fo $x^2 + 2x + 2 = 0$.

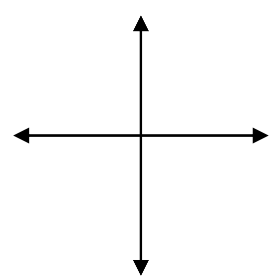
4.Sketch the graph of a quadratic function with vertex (2, 3) and a negative discriminant.



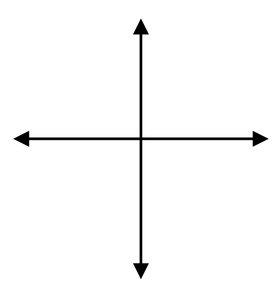
5. Sketch the graph of a quadratic function with axis of symmetry $x = -1$ and a positive discriminant.



6. Sketch the graph of a quadratic function with a maximum and a discriminant of zero.



7. Sketch the graph of a quadratic function that opens downward and has a negative discriminant.



Name: ____ Date: ___________________________

Extra Practice-Factoring Quadratic Equations

1. Factor.

a) $x^2 - x - 20$

b) $3x^2 - 30x + 63$

c) $-4x^2 - 12x - 8$

d) $\frac{1}{2}x^2 - \frac{1}{2}x - 6$

2. Factor.

a) $14x^2 + 3x - 5$

b) $3x^2 + 11x - 20$

c) $4x^2 + 7xy + 3y^2$

d) $6x^2 - 17x + 12$

3. Factor completely.

a) $12x^2 - 4xy - 8y^2$

b) $6x^2y + 27xy + 30y$

c) $140x^2 - 450xy + 250y^2$

d) $42x^3 + 77x^2y + 21xy^2$

4. Factor.

a) $x^2 - 49y^2$

b) $25x^2 - 9$

c) $x^2 - \frac{25}{4}y^2$

d) $(x + 1)^2 - (x - 7)^2$

5. Factor.

a) $(x - 1)^2 - 2(x - 1) - 35$

b) $6(2x + 1)^2 - 7(2x + 1) - 20$

c) $2(7x)^2 + 2(7x) - 24$

d) $8\left(\frac{1}{2}x^2\right)^2 - 6\left(\frac{1}{2}x^2\right) - 9$

6. Solve each quadratic equation by factoring. Verify your answer.

a) $x^2 - 2x - 15 = 0$

b) $2x^2 + 8x = 64$

c) $\frac{1}{2}x^2 - \frac{9}{2}x + 9 = 0$

d) $7x^2 - 35 = 0$

7. Solve each quadratic equation.

a) $6x^2 - 5x = 4$

b) $7x^2 = 34x + 5$

c) $5x^2 = 9x + 2$

d) $2x^2 + 9x = 18$

8. Determine the real roots of each quadratic equation.

a) $64x^2 - 169 = 0$

b) $18x^2 - 98 = 0$

c) $80x^2 = 5$

d) $(x + 1)^2 - 81 = 0$

9. Determine the real roots to each quadratic equation by factoring.

a) $6x^2 + 2x - 4 = 0$

b) $10x^2 - 45x + 20 = 0$

c) $18x^2 = 3x + 3$

d) $x^2 - \frac{5}{2}x - 21 = 0$

10. Solve each quadratic equation.

a) $9x^2 + 6x + 1 = 0$

b) $20x^2 - 60x + 45 = 0$

c) $x^2 + 5x + \frac{25}{4} = 0$

d) $1.6 - 5.6x + 4.9x^2 = 0$

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Extra Practice - Completing the Square

1. What value of c makes each trinomial expression a perfect square? What is the equivalent binomial square expression for each?

a) $x^2 - 10x + c$

b) $x^2 + 8x + c$

c) $x^2 - 12x + c$

d) $x^2 + 2x + c$

2. Write each function in vertex form by completing the square. Use your answer to identify the vertex of the function.

a) $y = x^2 + 2x - 4$

b) $y = x^2 - 6x + 13$

c) $y = x^2 + 8x + 6$

d) $y = x^2 + 24x + 54$

3. Convert each function to the form $y = a(x - p)^2 + q$ by completing the square.

a) $y = 3x^2 - 12x + 13$

b) $y = -2x^2 - 20x - 56$

c) $y = 6x^2 - 48x$

d) $y = -4x^2 - 56x - 196$

4. Write each function in vertex form. Determine the maximum or minimum of each function and the x-value at which it occurs. Then, sketch a graph of the function.

a) $y = x^2 + 6x + 4$

b) $y = 2x^2 - 16x + 31$

c) $y = -3x^2 - 12x - 7$

d) $y = -x^2 + 18x$

5. Convert each function to the form $y = a(x - p)^2 + q$. State the coordinates of the vertex, axis of symmetry, maximum or minimum value, domain, and range.

a) $y = x^2 + 10x + 16$

b) $y = -3x^2 - 6x + 3$

c) $y = 2x^2 + 30x + 117$

d) $y = 6x^2 - 4x + \frac{4}{3}$

6. If a farmer harvests his crop today, he will have 1200 bushels worth $6 per bushel. Every week he waits, the crop yield increases by 100 bushels, but the price drops 30¢ per bushel.

a) What quadratic function can be used to model this situation?

b) When should the farmer harvest his crop to maximize his revenue? What is the maximum revenue?

c) What assumptions are being made in using this model?

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Extra Practice - Solving Quadratic Equations by Completing the Square

1. What value of k makes each expression a perfect square?

a) $x^2 + 12x + k$

b) $x^2 - 20x + k$

c) $x^2 - 7x + k$

d) $x^2 + \frac{4}{5}x + k$

2. Complete the square to write each quadratic equation in the form $(x + a)^2 = b$.

a) $x^2 + 6x + 4 = 0$

b) $2x^2 - 16x + 10 = 0$

c) $-3x^2 + 15x - 2 = 0$

d) $\frac{1}{2}x^2 + 5x - 4 = 0$

3. Solve each quadratic equation, to the nearest tenth.

a) $(x - 4)^2 = 25$

b) $\left(x + \frac{1}{2}\right)^2 = \frac{1}{4}$

c) $(x - 0.1)^2 = 0.64$

d) $4(x + 7)^2 = 1$

4. Solve each quadratic equation. Express answers as exact roots in simplest form.

a) $x^2 + 2x - 2 = 0$

b) $x^2 - 5x + 3 = 0$

c) $x^2 + 0.6x - 0.16 = 0$

d) $x^2 - \frac{6}{7}x + \frac{9}{49} = 0$

5. Solve each quadratic equation by completing the square. Express answers in simplest radical form.

a) $4x^2 + x - 3 = 0$

b) $-3x^2 - 6x + 1 = 0$

c) $\frac{1}{4}x^2 + x - 5 = 0$

d) $-0.1x^2 + 0.6x - 0.5 = 0$

6. Solve each quadratic equation by completing the square. Express answers to the nearest hundredth.

a) $-2x^2 + 9x + 2 = 0$

b) $3x^2 - 3x - 1 = 0$

c) $\frac{1}{5}x^2 + 2x + 1 = 0$

d) $6x^2 + 3x - 2 = 0$

7. Two numbers have a sum of 22. What are the numbers if their product is 96?

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Extra Practice – The Quadratic Formula

1. Use the discriminant to determine the nature of the roots for each quadratic equation. Do not solve the equation.

a) $7x^2 + x - 1 = 0$

b) $3x^2 - 4x + 5 = 0$

c) $8y^2 - 8y + 2 = 0$

d) $3x^2 + 6 = 0$

2. Without graphing, determine the number of zeros for each quadratic function.

a) $f(x) = 3x^2 - 2x + 9$

b) $g(x) = 9x^2 - 30x + 25$

c) $h(t) = -4.9t^2 - 5t + 50$

d) $A(x) = (x + 5)(2x - 1)$

3. Use the quadratic formula to solve each quadratic equation. Express answers as exact values in simplest form.

a) $x^2 - 10x + 23 = 0$

b) $4x^2 - 28x + 46 = 0$

c) $9x^2 - 12x = -4$

d) $10x^2 - 15x = 0$

4. Use the quadratic formula to solve each quadratic equation. Express answers to the nearest hundredth.

a) $6x^2 - 5x + 1 = 0$

b) $-0.1x^2 + 0.12x - 0.08 = 0$

c) $-3x^2 + 5x + 4 = 0$

d) $\frac{x^2}{5} + \frac{2x}{3} - 1 = 0$

5. Determine the real roots of each quadratic equation. Express your answers as exact values.

a) $x^2 + 4x - 1 = 0$

b) $4x^2 - 4x - 7 = 0$

c) $8x^2 + 20x + 11 = 0$

d) $x^2 - 4x - 3 = 0$

6. Solve each quadratic equation using any appropriate method. Express your answers as exact values. Justify your choice of method.

a) $x^2 + 4x + 10 = 0$

b) $x^2 + 7x = 0$

c) $4x^2 + 20x + 25 = 0$

d) $(x + 4)^2 = 3$

e) $6x^2 + 2x - 1 = 0$

7. For the quadratic equation $2x^2 + kx - 2 = 0$, one root is 2.

a) Determine the value of k.

b) What is the other root?