Polynomials

Mrs. Kornelsen

The definitions and examples below, show and explain most of the terminology you will be using in the **algebra** section.

1. **Constant** - A constant is a number that always has the same value. One example is the number 8. In $2x^2 + 9$, the 9 is the constant. Other examples are:

   $3,\ {}^{1}/_{5},\ 3.5,\ -4,\ 10,\ x + \underline{9},\ 2x + \underline{6}$

2. **Variables** - A variable is a letter or letters that may be replaced (substituted) by numbers. Examples are:

   $a,\ x,\ x^2,\ y^3,\ m^2n^3,\ \underline{x} + \underline{z},\ 3\underline{x}$

3. **Expressions** - Expressions are formed by combining constants, variables or both by adding, subtracting, multiplying or dividing. Examples are:

   $ax + 3b,\ 5y,\ 2a - 7,\ x^2 + 2x - 1,\ {}^{3}/_{4}\, x^2y^6$

4. **Terms** - A term is each part of an expression separated by a plus or a minus sign. Examples are:

   $ax^2 + 5xy - 7x + 3$ ← (This expression has 4 terms.)

   $2x - 9$ ← (This expression has only 2 terms.)

5. **Factors** - Factors are numbers or variables forming a product. Examples are:

   ab ← (The factors are a and b.)

   $3x^2y$ ← (The factors are 3, x, x and y.)

6. **Coefficients (Numerical)** - Numerical coefficients are numbers associated with a product. The examples are underlined below.

   $(\underline{32})(a),\ \underline{-34}\, x^2,\ \underline{3.4}\, m^3z^2$

7. **Literal Coefficients** - Literal coefficients are the variables (letters) associated with a product. The examples are underlined below.

   $(32)(\underline{a}),\ -34\, \underline{x^2},\ 3.4\, \underline{m^3z^2}$

8. **Polynomials** - A polynomial is a monomial or the algebraic sum or difference of monomials. (See definitions below.)

   a) **monomial** - a monomial can consist of any or all of the following:

      1. any integer such as: 4, −3, 27, −198
      2. any variable with a positive integral power such as: y, a^2, m^5, $-a$, $-e$
      3. the product of integers and a variables such as: $2x$, $3a^2$, $-3x^2y^3$, $(-5y)(2x^4)$

   b) **binomial** - a binomial is the sum or difference of exactly 2 monomials as shown below:

      $a + b^2$, $3x^2 - 5y^2$, $7x - 2y$, $3x + 7$

   c) **trinomial** - a trinomial is the sum or difference of exactly 3 monomials as shown below:

      $ax^2 + bx + c$, $a - b + c$, $4m + 7n - 18$, $2x - 3y + 7x$

   d) We do not have special names for polynomials with 4, 5, 6, etc. terms. They are just called polynomials.

9. **Degree of Polynomials** - The largest exponent in a polynomial or the greatest sum of the exponents in any single term of a polynomial gives us the degree of a polynomial as the examples illustrate below.

   0 degree polynomials - 4, −5, $8x^0$

   1st degree polynomials - $3x$, $-6m$, $2y$, $3x + 5$, $8n^1 - 7$

   2nd degree polynomials - $8x^2$, $5y^2$, $9a^1b^1$, $13xy$, $4x^2 + 9x - 3$

   3rd degree polynomials - $6x^3$, $-5x^2y$, $2a^1b^1c^1$, $5xyz$, $4x^3 + 3x^2 + 8$, $3ab^2 + 9ab$

   4th degree polynomials - $3x^4$, $-4a^2b^2$, $-7a^4 + 9x^2 - 8y$, $9abcd$

10. **Standard Form of Polynomials (Descending Order)** - Descending order of polynomials or the standard form of polynomials is when we write the polynomial from its highest to its lowest degree as the examples illustrate.

    a) $-3x^3 + 8x^4 - 3x - 2x^2 - 7 = 8x^4 - 3x^3 - 2x^2 - 3x - 7$

    b) $3x + 2x^4 - 7x^5 + 9 = -7x^5 + 2x^4 + 3x + 9$

    c) $7b + 3a + 5c - 8x = 3a + 7b + 5c - 8x$ ← (In the event all variables have the same exponent or degree, we write the terms in alphabetical order.)

11. **Ascending Order of Polynomials** - Ascending order of polynomials is when we write the polynomial from its lowest to its highest degree as the examples illustrate. (Note that this is not used very often.)

    a) $5x^3 - 7x^5 + 3x^2 - 2 = -2 + 3x^2 + 5x^3 - 7x^5$

    b) $3a^2 + 7x - 8x^5 + 9x^3 = 7x + 3a^2 + 9x^3 - 8x^5$

Lesson One: Polynomial Definitions

Example 1:

For each expression, identify the number of terms and whether it is a monomial, binomial, trinomial or polynomial

a) $4xy + 3$

b) $7a^2 - 2ab + b^2$

c) $5x^2 + y^2 + z^2 - x - 6$

d) 13

Example 2:

What is the number of terms and the degree of each polynomial?

a) $4x^2 + 3$

b) $7a^2 - 2ab + b^2$

c) $5x + z - 6$

d) 7

Ordering Polynomials

Ascending order:

Descending order:

Example 3:

Order the following polynomials in ascending order of x

a) $x^4y + 4xy^3 - 2x^3y + 5y - 3$

b) $xy^3 + x^4y^2 - x^6$

Example 4:

Order the following polynomials in descending order of y

a) $xy^3 + 4x^2y - 5x^4y^7 + 10x$

b) $2y^8 - 3x^2y^3 + 4x^2y^4 - 3x$

A. How many terms are in each of the following expressions?

1. $2x + 3y$
2. $a + b - c$
3. $3x^2$
4. $(5x)(2y)$
5. $3x^2 + 7x + 5$
6. $-8x^3 + 5x^2 - 3x + 4$
7. $(3x)(-2x^2)$
8. $(6x^2) \div 2$

B. Circle the constant(s) in each of the expressions below.

1. $3x + 4$
2. $5x + 3y$
3. $8x^2 - 8$
4. $5a - 3b + 2c + 9$
5. $16x^2 - 5 + 3x$
6. $7x^2 - 3x + 4$
7. $x - 1.6$
8. $5 + 2x - 3x^2$

C. Circle the variable(s) in each of the expressions below .

1. $3a + 5b - 6$
2. $5x + 15$
3. $3x^2 - 4y^2 + 2z^2$
4. $a^2 + bc - 4c^2$
5. $x^2y^2 - 3xy + 5$
6. $-7y - 5z$
7. $14x - 10$
8. $a^3 + 2a^2 - 3a + 6$

D. Circle the numerical coefficient(s) in each expressions below.

1. $3xy$
2. $3x^2 + 4y^2$
3. $-4abc$
4. $2.6x^2 - 3.6y^2 + 7$
5. $14xy^3$
6. x
7. $4a^2b + 2ab^2$
8. $7x + y - z$

E. Label each polynomial below as to whether it is a monomial, a binomial or a trinomial.

1. $3x^2 + 5x - 7$
2. $ab + bc$
3. 7
4. $(x)(5)$
5. $(3x^2)(2x)(3)$
6. $7x^3 - 3x^2 + 5x$
7. $2a + 3b - 4c$
8. $(7x)(2y)$
9. $(3x^2)^3$
10. -7
11. $13x^2 - 5x$
12. $12x + 3$

F. State the degree of each of the following polynomials.

1. $3x^2$
2. a^8
3. $30a^4 + 15a^3$
4. $6x^2 - 3x + 2$
5. $x^3 - y^2$
6. $-5x^3 + 3x^2$
7. x^2y^3
8. $5x$
9. $3x^2y^3$
10. $3x^5 + 4x^2y^2 - 5y^3$

G. Complete each of the following statements.

1. Name the 2nd degree term in the expression $3x^3 + 5x^2 - 2x$. _____
2. Write any 4th degree binomial. _____
3. How many terms are in the expression $3 - 5xy + 3x^2y^3$? _____
4. Write out the constant(s) in the expression $3x^2 - 5xy + 2$. _____
5. Write the following polynomials in standard form:
   a) $7x^3 - 5x^6 + 2x^2 - 8x^5 + 23$ _____
   b) $8x^5 + 7x^4 - 6x^5y^3 + 9x^9$ _____
6. Write out the 2nd degree term in the polynomial $6x^3 + 3x^2y^2 + 5xy - 7$. _____
7. Write out the 3rd degree term in the polynomial $4x^2y^3 - 7x^3 + 3x^2$. _____
8. Write any 3rd degree trinomial in standard form. _____
9. Which is the numerical coefficient in the term $7x^2y^3z^4$? _____
10. Using the expression $(-8x^2 + 3x - 5)$, answer the following questions.
    a) It contains _____ terms, and is therefore called a _____.
    b) The second term contains _____ factors and they are _____.
    c) This polynomials is written in the _____ degree.
    d) The constant(s) in this expression is (are) _____.
    e) The numerical coefficient of the 2nd term is _____.
11. Write any 5th degree monomial. _____
12. Complete the polynomial by writing a 2nd degree term. $8x^7 + 4x^5 +$ _____ $+ 9x$
13. Complete the polynomial by writing a 3rd degree term. $7x^6 + 5x^5 -$ _____ $+ 6x$

Name: _____ Date: _____

Understanding Polynomials

1. Using the polynomial $4x^2 - y + 8$ what is/are the:
   a) constant: _____
   b) coefficient of x: _____
   c) coefficient of y: _____
   d) number of terms: _____
   e) exponent of x: _____

2. Identify each polynomial as a monomial, binomial or trinomial.
   a) $6 + 4b$
   b) $p^3 + 7p + 2$
   c) $\frac{5}{8}x^3y$
   d) $c + 3z - 9$

3. Answer the following for each polynomial

a) $a - 2b$	Is there a constant?
b) $x^2 + y^2 + z^2$	What are the coefficients?
c) 10	What is this classified as?
d) $5y - 4x + 100$	What is the constant?
e) $6x^3$	What is the exponent?
f) $\frac{5x^2y^4}{9z^3}$	What is the coefficient?

4. State the degree of each monomial.
   a) $3xyz$
   b) $21k$
   c) $9x^2y$
   d) $2a^2b^2c$
   e) 6
   f) $-45pq^4$

5. State the degree of each polynomial.
   a) $7k + 3$
   b) $31a + 4b$

c) $11x^2 + 8y$

d) $3a^2b + 4a^2b^3 + 5$

e) $-2a^6b^2 - ab + b^7$

f) $7k + h^3$

6. Arrange the terms in each polynomial in descending powers.

a) $2y + 5y^3 + 16y^2$

b) $-3x + x^4 + 5x^3 - 9$

c) $2x^2y - 3xy^3 + x^3$

d) $3y^2x^3 - 4x^2 - 6y^2x - 8$

Lesson Two: Equivalent Expressions

Example 1:

For each expression, identify the coefficient, the variable(s), and the exponent of each variable

b) $3w$ b) a^2 c) $-4xy$ d) $-g$

Example 2:

Identify the like terms in each group

b) $5b^2$ $3cb$ $-2b$ $7c$ $6b$

c) $3x^2$ $4xy$ $-2x^2$ $7x^2$ $\frac{1}{2}y$

d) $3pq$ 11 $-4q^2$ -3 pq

Example 3:

Combine like terms in each expression

a) $4x - 2x + 3 - 6$

b) $2x^2 + 3x - 1 + x^2 - 4x - 2$

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

Lesson Three: Adding Polynomials

Example 1:

Add $3x - 4$ and $2x + 5$. Simplify your answer by combining like terms

Example 2:

Add $2x^2 + 5x - 3$ and $-x^2 + x - 6$

Example 3:

Add $(-3x^2 + 6x - 5) + (4x^2 + x - 9)$

Lesson Four: Subtracting Polynomials

Opposite Polynomial:

Example 1:

What is the opposite polynomial for each of the following?

a) $3x$ b) -2 c) $4x-1$ d) a^2-3a+2

Example 2:

Subtract $2x+3$ from $3x-4$. Simplify your answer by combining like terms

Example 3:

$(4x - 5) - (2x + 3)$

Example 3:

$(3x^2 + 7x - 8) - (-x^2 + x - 1)$

Lesson Five: Multiplying Monomials

Think back to the exponent laws:

When multiplying powers, _____ exponents

Ex. $3x \cdot 6x^2$

When simplifying a power to a power, _____ exponents

Ex. $(2y^2)^3$

Example 1:

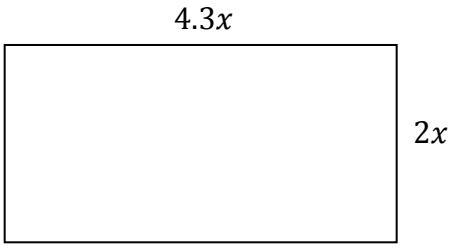
Use alge-tiles to determine the product of $(5x)(2x)$

Example 2:

Use exponent laws to determine the product of $(3x)(2y)$

Example 3:

What is an expression for the area of the rectangle?



Lesson Six: Multiplying Polynomials by Monomials

Think back to the distributive property:

Ex. $6x(2x + y)$

Example 1:

Use alge-tiles to determine the product $(3x)(2x + 4)$

Example 2:

Use exponent laws to determine the product of $(2x)(3x - 5)$

Example 3:

The dimensions of a rectangular gym floor are represented by the expressions $4x$ and $5x - 3$. What is a polynomial expression for the area of the gym floor? Write the expression in simplified form.

Lesson Seven: Dividing Monomials

Think back to the exponent laws:

When dividing powers, _____ exponents

Ex. $\frac{6x^2}{3x}$

Example 1:

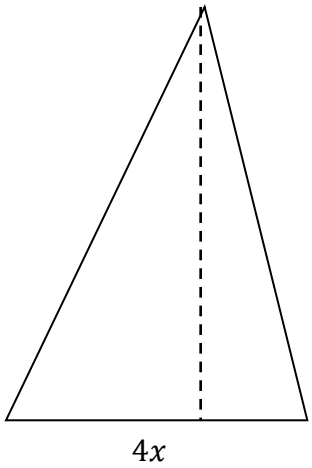
Use alge-tiles to determine the quotient of $\frac{-10x^2}{2x}$

Example 2:

Use exponent laws to determine the quotient of $\frac{8xy}{4x}$

Example 3:

The area of a triangle is given by the expression $18x^2$. The base of the triangle is represented by $4x$. What is the height of the triangle in terms of x?



Lesson Eight: Dividing Polynomials by Monomials

Example 1:

Use alge-tiles to determine the quotient of $\frac{6x^2-8x}{2x}$

Example 2:

Use exponent laws to determine the quotient of $\frac{3x^2+6x}{3x}$

Example 3:

Use exponent laws to determine the quotient of $\frac{4x^3-6x+12x^2}{2x}$