Applied 40S

Unit 1 – Investments

Mrs. Kornelsen

Teulon Collegiate Institute

Learning checklist – Investments

Learning increases when you have a goal to work towards. Use this checklist as guide to track how well you are grasping the material. In the center column, rate your understand of the topic from 1-5 with 1 being the lowest and 5 being the highest. Be sure to write down any questions you have about the topic in the last column so that you know what you have yet to learn.

Outcomes	**Understanding**	**Questions?**
Solve problems that involve compound interest in financial decision making		
- Explain advantages and disadvantages of compound interest and simple interest		
- Identify situations involving compound interest		
- Graph and compare the total interest paid or earned for different compounding periods		
- Graph and describe the effects of changing the value of one of the variables in a situation involving compound interest		
- Solve a contextual problem involving compound interest.		
Analyze an investment portfolio in terms of interest rate, rate of return and total return.		
- Determine, using		

technology, the total value of an investment when there are regular contributions to the principal		
- Graph and compare the total value of an investment with or without regular contributions		
- Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule.		
- Determine, using technology, possible investment strategies to achieve a financial goal.		
- Explain the advantages and disadvantages of long-term or short-term investment options.		
- Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term.		
- Determine and compare the strengths and weaknesses of two or more portfolios.		

Unit 1: Investing Money

Vocabulary

Investment: Putting money into something with the expectation of gain (interest).

Savings Accounts: One of the easiest and safest ways to save money by lending it to a bank, trust company or credit union.

Tax Free Savings Accounts: A savings vehicle whereby income earned within a TFSA will not be taxed in any way throughout an individual's lifetime. In addition, there are no restrictions on the timing or amount of withdrawals from a TFSA, and the money withdrawn can be used for any purpose. However, you can only put a certain amount of money each year into a TFSA.

Guaranteed Investment Certificates (GICs): have terms ranging from one to five years. The interest rate is guaranteed and the money is usually locked in until maturity. Minimum deposits may be required.

Canada Savings Bonds: Issued by the federal government in November of each year for a limited time and vary in denominations from $100 to $10000. The government will redeem them at face value on any business day, even before maturity date. These bonds are very secure.

Stocks: When you own shares of stock, you become part-owner of a company. If the company does well, the value of your stock should go up over time. If the company does not do well, the value of your investment will decrease.

Mutual Funds: Funds made up of a group of selected stocks and bonds. Investors buying shares in a mutual fund are, in a way, investing in the investment company that manages the fund. Mutual funds are not risk-free and their values rise and fall along with the securities of the fund.

Real Estate: Property of a permanent nature; for example, land and buildings. For most people, investing in real estate means buying a home of their own.

Interest: The amount of money earned on an investment or paid on a loan (fee). Interest rates are communicated as a percent for a time period. A given % is assumed to be annual unless otherwise stated.

Term: The contracted duration of an investment or loan.
Fixed Interest Rate: An interest rate that is guaranteed not to change during the term of an investment or loan.

Principal: The original amount of money invested or loaned.

Maturity: The contracted end date of an investment or loan, at the end of the term.

Rate of Return: The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

1.1 Simple Interest

Simple Interest: The amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate.

The formula for calculating simple interest is:

$I = Prt$ where: I = interest
P = principal, or amount of money initially invested
r = the interest rate (as a decimal)
t = number of years

Future Value: The amount, **A,** than an investment will be worth after a specified time period. The formula for calculating the amount, **A,** that an investment will be worth after a specified time period is:

$$A = P + Prt$$

Example 1: Marty invested in a $2500 GIC at 2.5% simple interest, paid annually, with a term of 10 years.

a. How much interest will accumulate over the term of Marty's investments?

b. What is the future value of his investment at maturity?

c. How would an interest rate of 3.5% change the future value of Marty's investment?

Example 2: Sunni invested $15 000 in a savings account. Sunni earned a simple interest rate of 8%, paid semi-annually on her investment. She intends to hold the investment for 4.5 years, when she will withdraw all the money to buy a car. Determine the value of the investment at each half year until she withdraws the money.

Year	Value ($)

a. Use a table of values.

P =

r =

t =

b. Verify using the formula. $A = P + Prt$

c. Use a graph.

Compare this with the equation $y = mx + b$.

The y-intercept or A-intercept is _____. The slope is _____.

Example 3: Grant invested $25 000 in a simple interest CSB that paid interest annually.

a. If the future value of the CSB is $29 375 at the end of 5 years, what interest rate does the CSB earn?

b. What is Grant's rate of return after 5 years? $\left(\text{Rate of Return} = \dfrac{\text{interest earned}}{\text{principal}}\right)$

c. Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

1.2 Exploring Compound Interest

Compound Interest: The interest that is earned or paid on both the principal and the accumulated interest.

Example 1: Comparing Simple and Compound Interest
Both Evan and Rena received a $1000 prize in a story-writing contest.

- Evan bought a $1000 simple interest GIC with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Rena bought a $1000 compound interest GIC with her prize money. It also has a 5-year term and earns 3.6% paid annually.

a. How do the future values of Evan's and Rena's investments compare at maturity?

Use a table to show this comparison.

Evan

Year	Value of Investment at Start of Year ($)	Simple Interest Earned Each Year ($)	Value of Investment at the End of Year ($)
0			
1			
2			
3			
4			
5			

Equation for calculating the future value of a simple interest investment: _____

Rena

Year	Value of Investment at Start of Year ($)	Simple Interest Earned Each Year ($)	Value of Investment at the End of Year ($)
0			
1			
2			
3			
4			
5			

The formula for calculating compound interest is:

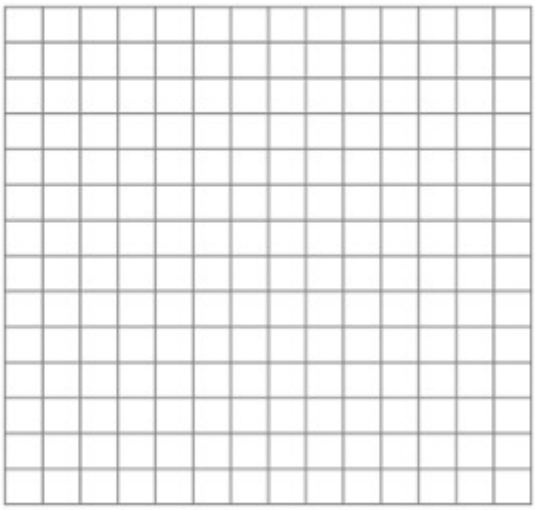
$A = P\left(1 + \frac{r}{n}\right)^{nt}$ where: A = future value

P = principal or present value
r = interest rate per compounding period
n = compounding period
t = number of years

Equation for calculating the future value of a compound interest investment:

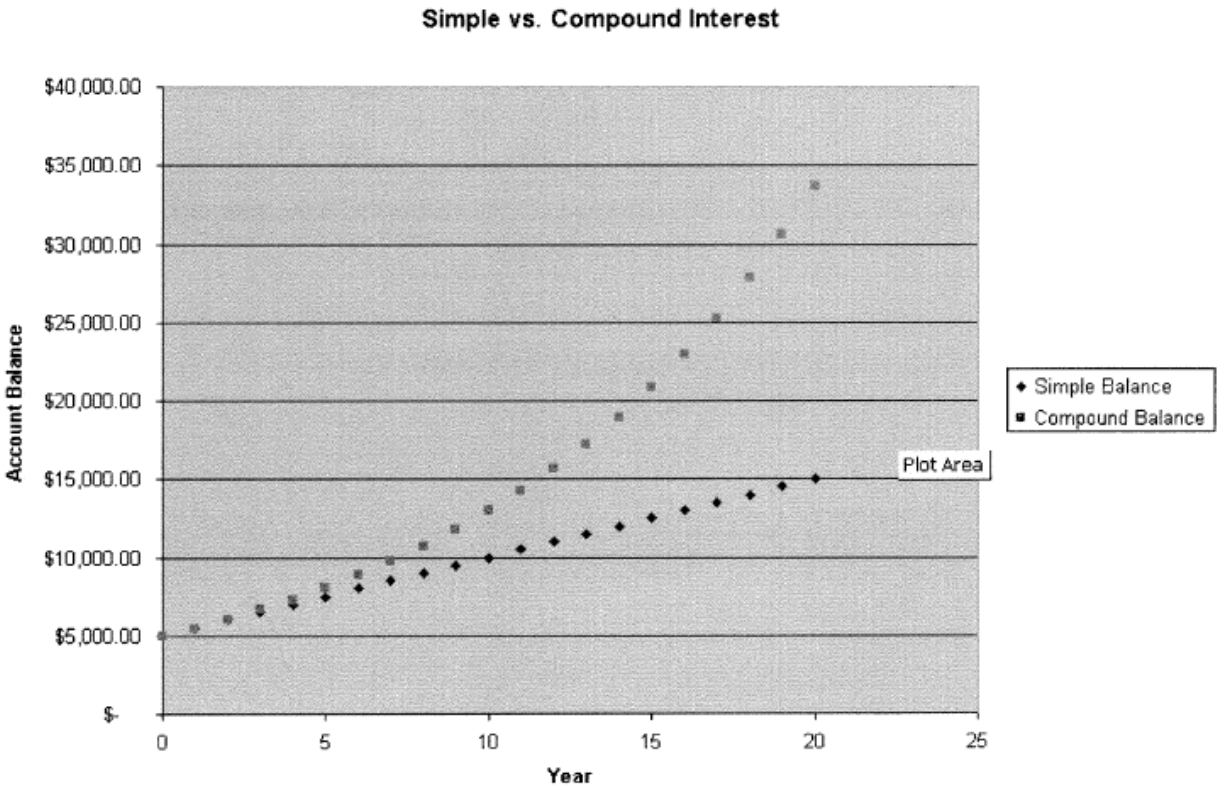
_____

b. Graph both investments on the same coordinate grid. How are the shapes of the graphs different? Explain why.



Main Idea:

- Compound interest grows faster than simple interest and is a non-linear function. (Compound interest is an *exponential function*.)



Compounding Period: The time over which interest is determined.

Annually: yearly
Semi-annually: every 6 months or 2 times per year
Quarterly: every 3 months or 4 times per year
Monthly: 12 times per year
Weekly: 52 times per year
Daily: 365 times per year

Example 2: Yvonne earned $4300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% compounded annually. She decided to invest in a CSB, instead of keeping her money in a savings account, because the CSB will earn more interest. What is the future value of Yvonne's investment after 10 years?

Example 3: Matt has invested a $23 000 inheritance in an account that earns 13.6% compounded semi-annually. The interest rate is fixed for 1- years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

a. What is the future value of the investment after 5 years? What is the future value after 10 years?

b. Compare the principal and the future value at 5 years and 10 years. What do you notice?

Example 4: Celine wants to invest $3000 so that she can buy a new car in the next 5 years. Celine has the following investment options:

A. 4.8% compounded annually
B. 4.8% compounded semi-annually
C. 4.8% compounded monthly
D. 4.8% compounded weekly
E. 4.8% compounded daily

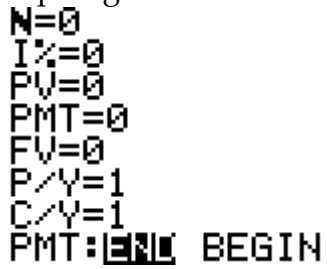
Compare the interest earned by each of these options for terms of 1 to 5 years.

	A	B	C	D	E
Principal $	3000	3000	3000	3000	3000
Interest Rate	0.048	0.048	0.048	0.048	0.048
Periods Per Year					
Value at End of Year 5					

Rule of 72: A formula that estimates the doubling time of an investment; 72 is divided by the annual interest rate as a percent to give the number of years required to double your investment.

$$\frac{72}{interest\ rate\ as\ a\ \%} = \text{number of years for investment to double}$$

Graphing Calculator: Press 'APPS', '1:Finance...' and '1:TVM Solver'.



N =

I% =

PV =

PMT =

FV =

P/Y =

C/Y =

PMT: END BEGIN

Note:

- The calculator displays either positive or negative values for PV, PMT, and FV.
- A **negative value** indicates the amount is **invested or paid out by us.**
- If we input any 6 of the first 7 values, the calculator can solve for the missing value using **ALPHA SOLVE (ENTER).**

Example 5: Both Justine and Kris invested $5000 by purchasing CSBs. Justine's CSB earns 8%, compounded annually, while Kris's CSB earns 9% compounded annually.

a. Estimate the doubling time for each CSB.

b. Verify your estimates by determining the double time for each CSB.

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

c. Estimate the future value of an investment of $5000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

Main Ideas:

- Use an exact value for $\frac{r}{n}$ → no rounding!
- Remember to follow BEDMAS when entering the formula in your calculator
- The total compounding interest can be calculated by taking the final amount minus the principal: $I = A - P$
- The **Rule of 72** tells you the number of years it takes for an investment to double

1.3 Compound Interest: Present Value

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ where: A = future value
P = principal or present value
r = interest rate per compounding period
n = compounding period
t = number of years

Present Value: The amount that must be invested now to result in a specific future value in a certain time at a given interest rate.

How could we use this formula to solve for the present value instead of the future value?

Instead, let's use TVM Solver on the calculator!

Example 1: Ginny is 18 years old. She has inherited some money from a relative. Ginny wants to invest some of the money so that she can buy a home in Milk River, Alberta, when she turns 30. She estimates that she will need about $270 000 to buy a home.

a. How much does she have to invest now, at 6.5% compounded annually?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

b. What is the ratio of future value to present value for Ginny's investment?

Example 2: Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that $40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a. How much money should they have invested?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

b. How much interest will they earn over the return of the investment?

Example 3: Laura has invest $15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least $50 000 by the time her newborn enters university in 18 years. What interest rate, compounded annually, will result in a future value of $50 000? Round your answer to two decimals.

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

1.4 Investments Involving Regular Payments

Example 1: Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6 month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

It's easiest to use the TVM Solver for this:

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

Example 2: Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

a. What was their future value?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

b. What principal amount did Blake invest 5 years ago?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

c. Who earned more interest? Why?

Example 3: Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value if $10 059.07. What annual rate of interest does Jeremiah's investment earn?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

Example 4: Celia wants to have $300000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

a. What regular payments must Celia make at the end of each year to meet her goal of $300 000?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

b. How much interest will she earn over the 20 years?

Example 5: On Luis's 20th birthday, he started making regular $1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have more than $18 000?

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

Main Idea:

- Small deposits over a long time can have a greater future value than large deposits over a short term because there is more time for compound interest to be earned

1.5 Solving Investment Portfolio Problems

Investment Portfolio: One or more investments held by an individual investor or by a financial organization.

Example 1: Phyllis started to build an investment portfolio for her retirement. She purchased a $500 CSB at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. Three years ago, she also purchased a $4000 GIC that earned 6% compounded monthly.

a. What is the value of Phyllis's portfolio 10 years after she started to invest?

- CSBs

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

- GIC

N =
I% =
PV =
PMT =
FV =
P/Y =
C/Y =
PMT: END BEGIN

a) Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio because interest rates went up and invested all the money in the savings account. About how long will it take her to double all her money?

Example 2: Jason and Malique are each hoping to buy a house in 10 years. They want their money to grow so they can make a substantial down payment.

Jason's portfolio:

- A 10-year $2000 GIC that earns 4.2% compounded semi-annually
- A savings account that earns 1.8%, compounded weekly, where he saves $55 every week
- A 5-year $4000 bond that earns 3.9% compounded quarterly, which he will reinvest in another bond at an interest rate of 4.1%

Malique's portfolio:

- A TFSA that earns 2.2%, compounded monthly, and has a current balance of $5600
- The purchase, each year, of a 10-year $500 CSB that earns 3.6%, compounded annually
- A savings account that earns 1.6%, compounded monthly, where she saves $200 a month

a. In 10 years, whose portfolio will have the greater rate of return on investment?

$$\text{Rate of Return} = \frac{\text{Interest Earned}}{\text{Total Invested}} \text{ x } 100$$

Jason's Portfolio	Malique's Portfolio
Total Invested:	Total Invested:
Portfolio Value: N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN	Portfolio Value: N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN

N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN Total Portfolio:	N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN N = I% = PV = PMT = FV = P/Y = C/Y = PMT: END BEGIN Total Portfolio:
Interest Earned:	Interest Earned:
Rate of Return:	Rate of Return:

b. What recommendations could you make to help either Jason or Malique increase his or her rate of return?

Main Ideas:

- Rate of return compares investment portfolios
- Investments that involve greater principal amounts invested or greater regular payment amounts when contracted tend to offer higher interest rate.
- The factors that contribute to a higher rate of return are time, interest rate, and compounding and payment frequency.

Investing Money Test Review

1. $420 is invested at 2.35% simple interest for 5 years. Determine the interest earned.
    a. $469.35
    b. $49.35
    c. $51.72
    d. $471.72
2. $10 000 is invested at 9.2% simple interest for 10 years. Determine the future value of the investment.
    a. $9200
    b. $24 111.62
    c. $19 200
    d. $14 111.62
3. $5000 is invested at 6.7% simple interest for 20 years. Determine the interest earned.
    a. $6700
    b. $11700
    c. $335
    d. $33 500
4. According to the Rule of 72, how long would it take for $400 to grow to $800 at 5.6% interest, compounded monthly?
    a. 12.9 years
    b. 403 years
    c. 1 year
    d. None is correct.
5. How long would it take for $500 to grow to $1200 at 6.6% interest, compounded monthly?
    a. 2 years
    b. 159.6 years
    c. 10 years
    d. 13.3 years
6. Determine the present value of a 5-year GIC with an interest rate of 6.3%, compounded monthly, if the future value is $10 000.
    a. $7303.90
    b. $2696.10
    c. $13 691.31
    d. $3691.31

7. Determine the present value of a 10-year CSB with an interest rate of 4.9%, compounded semi-annually, if the future value if $5000.
    a. $41 918.72
    b. $3081.28
    c. $8113.52
    d. $3113.52

8. For 8 years, regular monthly payments of $500 are deposited into an account in which interest is compounded monthly. If the final value of the account is $60,000, what was the interest rate?
    a. 0.54%
    b. 4.69%
    c. 2.5%
    d. 5.45%