Applied 30S Unit 1 – Quadratic Functions



Mrs. Kornelsen

Teulon Collegiate Institute

Learning checklist - Quadratics

Learning increases when you have a goal to work towards. Use this checklist as guide to track how well you are grasping the material. In the center column, rate your understand of the topic from 1-5 with 1 being the lowest and 5 being the highest. Be sure to write down any questions you have about the topic in the last column so that you know what you have yet to learn.

Outcome	Understanding	Questions
Demonstrate an understanding of the characteristics of		
quadratic functions, including		
- vertex		
- intercepts		
 domain and range 		
 axis of symmetry 		
Determine, with technology, the intercepts of the graph of		
a quadratic function or the		
roots of the corresponding quadratic equation.		
Explain the relationships among the roots of an equation,		
the zeros of the corresponding		
function, and the x -intercepts of the graph of the		
function.		
Explain, using examples, why the graph of a quadratic		
function may have zero, one, or two		
x intercepts.		
Express a quadratic equation in factored form, using the		
zeros of a corresponding function		
or the x -intercepts of its graph.		
Determine, with technology, the coordinates of the vertex		
of the graph of a quadratic		
function.		
Determine the equation of the axis of symmetry of the		
graph of a quadratic function, given		
the x -intercepts of the graph.		
Determine the coordinates of the vertex of the graph of a		
quadratic function, given		
the equation of the function and the axis of symmetry, and		
determine whether the		
y -coordinate of the vertex is a maximum or a minimum.		
Determine the domain and range of a quadratic function.		
Sketch the graph of a quadratic function.		
Solve, with technology, a contextual problem involving		
data that is best represented by		
graphs of quadratic functions, and explain the reasoning.		
Solve a contextual problem that involves the		
characteristics of a quadratic function.		

Factoring Polynomial Expressions - Review

Monomial by Monomial:

$(4x)(7x^2) =$	=		

 $(-6m^2n^3)(-7mn^2) =$ _____

Monomial by Binomial: Distributive Property

3x(x - 2) =		
· · · ·		
$5y^2(x^2 - y) =$	 	

Monomial by Trinomial:

abc(3a + 4b - 2c) =_____

 $(2y^2 + 3y - 1)(4y) =$ _____

Binomial by Binomial (F. O. I. L.):

(2x - y)(3x + y) =_____

=_____

=_____

(x + 6)(x + 8) =_____



Binomial Squared: (F. O. I. L.)

$(x + y)^2 =$	 	
=		
$(x + 5)^2 = $		
_		
-		
$(2x - y)^2 =$		
=		
$-4(3x + y)^2 =$		
=		
=		

Prerequisite Skills: Simplify the following

- a) $7x^2 3x + x^2 x$ b) (4x - 3)(x + 7)c) $(2x - 5)^2$ d) $(x - 1)^2 - (2x + 3)(x - 4)$ e) 3(2x - 7) - 4(x - 1)f) 5x(3x - 2)
- **g)** (4x 3)(2x + 5) **h)** $(5x 4)^2$

Pre-Calculus 20S Name:_____

Binomial Products			
1.	(a + 3)(a + 2) =	=	
2.	(x - 1)(x - 2) =	=	
3.	(2 + k)(3 + k) =	=	
4.	(c - 5)(c - 3) =		
5.	(y - 4)(y + 6)	=	
6.	(y + 5)(t - 1) =		
7.	(3 - b)(4 + b) =	=	
8.	(6v + 3)(v + 2)	=	
9.	(5+3x)(2+x)	=	

10.	(y - 5)(2y - 2) =	
11.	(m + 3)(3m - 2) =	
12.	(2a + 3)(3a + 2) =	
13.	(4 + 3p)(3 - 4p) =	
14.	2(a + 3)(a + 2) =	
15.	-3(y - 7)(y + 5) =	
16.	(a - 3)(a + 3) =	
17.	(5a + 3)(5a - 3) =	
18.	$(a^2 - 6)(a^2 + 6) =$	
19.	(a + 4)(a + 4) =	

20.	$(x - 3)^2 =$
21.	$(2c + 5)^2 =$
22.	$(4a - 3b)^2 =$
23.	$(y + 3)(y^2 - 7y + 5) =$
24.	$(a^2 - 3a + 11)(a - 3) =$
25.	$(2x - 3)(2x^2 - 3xy + y^2) =$

Exercise 1: Factoring Review (Common and Simple Trinomial)

Factoring is the reverse process of ______. The better you are at multiplying, the better you will be at factoring.

Multiplication	Factoring
$5x(x-2y) = 5x^2 - 10xy$	$9x^2 - 15x = 3x(3x - 5)$
$(x-3)(x+5) = x^2 + 2x - 15$	$x^2 + 8x + 15 = (x + 3)(x + 5)$

Common Factoring:

- When factoring, always begin by looking for any numbers or variables that is (are) common to (appears in) every term. It could be a number, a variable or both.
- Place this common factor in front of the parentheses, with the remaining polynomial inside the parentheses.
- Once this is done, the same number of terms as in the original question should be inside the brackets.

Examples:	8x - 32y =() b - b ²	$r^{3}c = $ (
)	Common facto	or Remaining factor	Common factor
Remaining fac	tor	0	

$3x^3 - 6x^2y + 9xy^2 =$	()
--------------------------	----

Common factor Remaining factor

• Factoring can always be quickly and easily checked by ______ the _____ together to see if the result is the same as the original polynomial.

Trinomial Factoring:

- When factoring a trinomial, you need to ask yourself: "*Where did it come from*?" Trinomials are most commonly the result of multiplying two binomials using ______.
- For now, we will consider only trinomials where the lead coefficient in front of the "x²" is one. If it is not 1, try to make it one by factoring out a common value amongst all terms.

Example:





- Once you have factored out any common terms/variables, the first terms of each binomial will simply be the square root of the first term of the trinomial.
- To determine the last terms of each binomial, we need to first identify the factors of the last term of the trinomial. In this case, the factors of 6 are: _____; and _____;
- Next, determine which pair of factors either adds up to or subtracts to get the _____ term.
 (In this case: _____). These factors become the last terms of the binomial
- () Therefore, $x^2 + 5x + 6$ factors to ()()

Check your factoring by multiplying using FOIL!

• There are $\underline{4}$ *types* of simple trinomials:

1.	$x^2 + 8x + 12$	Sign in the is	in both parentheses.	$\Rightarrow (x+2)(x+6)$
2.	x ² - 8x +16	Sign in the is	s in both parent	heses. \Rightarrow (x - 4)(x - 4)
3.	x ² - 5x - 24	One middle sign is	, one is	$\ \Rightarrow (x+3)(x-8)$
4.	$x^2 + 3x - 54$	One middle sign is	, one is	$\ \Rightarrow (x+9)(x-6)$
<u>Exan</u>	nples: Facto	r the following trinomials:		
1. ① C ② F	$x^2 + 9x + 18$ Common terms?	Y/N The first term: _		
3 W	hich of these pa	irs add/subtract to:		
@ ≓	> ()()		
Mor	e Examples:			
2.	$x^2 - 2x - 15 =$		④ ⇒ ()()
3.	$x^2 - 6x + 5 =$		④ ⇒ ()()
4.	a² - 3a + 2 =_)()
5.	a² + 10a + 24	=	④ ⇒ ()()
6.	$x^2 - 4x - 60 =$		④ ⇒ ()()
7.	$x^2 + 8x =$			_)()

8.	b ² + b - 12 =	_④ ⇒ (_)()
9.	$2x^2 + 8x + 6 =$			
	=	④ ⇒()()
10.	xy ² + xy - 13y =			
	=	④ ⇒ (_)(_)
11.	$x^2 - 6x + 9 =$	_ ④ ⇒ ()() = (_)

	Pre-Calculus 20S	Name:
		Factoring Using F.O.I.L.
1.	$x^2 + 10x + 24 =$	
2.	p ² + 8p + 16 =	
3.	$c^2 - 17x + 72 =$	
4.	$a^2 - 7a + 6 =$	
5.	$x^2 - 14x + 49 =$	
6.	$d^2 - 4d - 45 =$	
7.	$m^2 - 15m + 50 =$	
8.	k4 + 11k2 + 30 =	
9.	$x^2 + 5x + 5 =$	
10.	$a^2 - 5a + 4 =$	
11.	y ² + 12y + 35 =	
12.	$z^2 - 12z + 35 =$	
13.	$v^2 + 2v - 35 =$	
14.	$d^2 - 2v - 35 =$	
15.	$2x^2 + 12x + 10 =$	
16.	4y + 10 =	
17.	6m ² + 9m =	
18.	$4t^3 - 6t^2 =$	
19.	$3p^3q^2r^2 - 4p^2q^2r =$	
20.	5a ² + 10ab – 15b ² =	

Exercise 1: Factoring $ax^2 + bx + c$

• Recall: Factoring is the inverse of multiplication.

Example: Factor $15x^2 + 48x + 36$

Product-Sum-Factor (PSF) Method							
0	ALWAYS begin factoring by checking for common factors within each term.						
2	From the remaining trinomial, calculate the product of the first and last coefficients.						
3	List all of the factors of this product. (List them as pairs)						
	④ ④ From these <i>pairs</i> of factors, identify which pair, when <u>added</u> together, result in the middle term of the trinomial.						
9	Re-write the trinomial with the original first and last terms. In between these two terms, insert two new terms using the pair of factors from step ④. When inserting these two new terms, think about which two should go together. That is, which two coefficients have a common factor?						
©	Now factor the first two and the last two terms. (Notice how a common factor emerges.)						
0	Now factor again.						
8	Check your answer by expanding (multiplying using FOIL)						

- 0 Any common terms?
- ^② Product of 1st & last coefficients?
- ③ Product factors (pairs)?

In summary, to factor a trinomial of the form $ax^2 + bx + c$, look for two integers with a sum of b and a product of ac.

④ Which pair = Sum of middle term?

Exan	ples: Factor the following fully, <i>if possible</i> :
1.	$3x^2 + 17x + 10 =$
2.	$3y^2 - 10y + 8 =$
3.	$8a^2 + 18a - 5 =$
4.	$2c^2 + 2c - 3 =$
5.	$5x^2 - 20xy + 20y^2 =$
6.	$6b^4 + 7b^2 - 10 =$
7.	$2d^3 + 7d^2 - 30d =$
8.	$12 + 18e + 8e^2 =$
9.	$9f^2 - 15f - 4 =$
10.	$10g^2 - 3gh - h^2 =$
11.	$6k^2 + 14km - 12m^2 =$
10	$24v^2z + 28vwz - 26zv^2 - 26$

12. $24x^2z + 38xyz - 36zy^2 =$

Exercise 1: Factoring Difference of Squares (& *Perfect* Squares)

A) Difference of Squares: $ax^2 - by^2$

- A difference of squares has 3 main features:
 - 1. The first term is a *perfect square*.
 - 2. The second term is a *perfect* square.
 - 3. They are separated by a *minus* sign.
- The middle term is missing because it is **0**.

Eg.
$$x^2 - 0xy - 16y^2$$

- Factoring a perfect square *binomial* results in 2 almost identical factors, that differ only in the middle negative and positive signs. **Note:** You can't factor if both middle terms are "+".
- To factor a difference of squares:
 - **①** Remember to always begin factoring by looking for a **common** factor.
 - ^② The first term of the factors comes from the square root of the first term.
 - **③** The second term of the factors comes from the square root of the second term.
 - Place a <u>negative</u> sign in one parentheses and a <u>positive</u> in the other.
 - **⑤** Check the result by using FOIL.

Example from above: $x^2 - 16y^2 = ($ _____)(____)

Examples:

- **1.** $x^2 9 = ($ ____)(____)
- **2.** $225b^2 a^2 = ($ ____)(____)
- **3.** $49 + x^2 = (__)(__)$
- 4. $-y^2 + 36 = (__)(__)$

=

5. $3x^3 - 48x =$

x⁴ - 16 =

6.

Example: $x^2 - 16y^2$

B) Perfect Square *Trinomials*: $ax^2 \pm bxy + cy^2$

- A *perfect* square trinomial has 4 main features:
 - 1. The first term is a *perfect square*.
 - 2. The third term is a *perfect square*.
 - 3. The *sign* of the 3^{*rd*} term is always *positive*.
 - 4. The *middle* term is always *twice* the <u>sum/difference</u> of the <u>outside/inside</u> product of the binomials.
- Factoring a perfect square *trinomial* results in two **identical binomials**.

```
• To factor a perfect square trinomial:
```

- **O** Remember to ALWAYS begin factoring by looking for a **common factor**.
- The first terms of the factors come from taking the square root of the first term of the trinomial.
- The 2nd terms of the factors comes from taking the square roots of the 3rd term of the trinomial.
- The middle signs of the factors will always be either positive or negative, depending on the sign of the <u>middle</u> term of the trinomial.

Remember: The last term of the trinomial is +. $(+ \times + = +$ or $- \times - = +)$ **⑤** Check the result by using FOIL.

Example: Factor: $x^2 - 8xy + 16y^2 = ($)()

Check:

Examples: Factor the following trinomials (1-5) fully, if possible.

- **1.** $49 + 14x + x^2 = (__)(__)$
- **2.** $5b^3 40b^2 + 80b =$ (_____)
 - = _____)(_____)
- 3. $4p^2 + 20pq + 25q^2 =$
- 4. $16y^2 + 24y 9 =$
- 5. $121x^2 22x + 1 =$
- 6. The *volume* of a *rectangular* prism is represented by $2x^3 24x^2 + 72x$. What are possible *dimensions* of the prism?=_____)

Example: $x^2 - 8xy + 16y^2$

Practice - Factoring (ax² + bx + c and perfect squares)

Name: _____

Factor the following polynomials completely.

1.	5a² + 15a – 20	2.	2b ² + 2b - 112
3.	9c ² + 54c +45	4.	3d² + 15d - 108
5.	7e ² - 84e - 196	6.	g² + 5gh + 6h²
7.	$x^2y^2 + 5xy + 6$	8.	5km² – 40km + 35k

9. $f^4 + 9f^2 + 14$ 10. $2n^4 + 16n^2 + 30$

11. $6x^3y - 7x^2y^2 - 3xy^3$ 12. $4s^2 - 21st - 18t^2$

13. $p^2 - 64$ 14. $9r^2 - 81$

15. $16 - 49u^2$ 16. $25v^2 - 169w^2$

17. $1 - 16x^4$ 18. $y^4 - 16$

19. $63a^2b - 28b$ 20. $81x^2 - (3x + y)^2$

21. $(5m - 2)^2 - (3m - 4)^2$ 22. $(3y + 8z)^2 - (3y - 8z)^2$

1.1 Properties of Graphs of Quadratic Functions

A quadratic function can be written in the form

$$y = ax^{2} + bx + c$$
 or $f(x) = ax^{2} + bx + c$

where a, b, and c are real numbers.

Vocabulary of Quadratic Functions:

Definition: Quadratic Function	Diagram/Example:
Definition: Quadratic Equation	Diagram/Example:
Definition Parabola	Diagram /Evampla:
	Diagram/ Example.
Definition: Vertex	Diagram/Example:
Definition: Axis of Symmetry	Diagram/Example:

Definition: Maximum Value of a Quadratic	Diagram/Example:
Definition: Minimum Value of a Quadratic	Diagram/Example:
Definition: Domain	Diagram/Example:
Definition: Range	Diagram/Example:
Definition: <i>x</i> -intercept or zeros	Diagram/Example:
Definition: <i>y</i> -intercept	Diagram/Example:

Example 1: Identify Characteristics of Graphs of Quadratic Functions

For each of the following graphs:

- a. Determine the equation for the axis of symmetry and draw it on the graph.
- b. Determine the coordinates of the vertex of the parabola.
- c. State the domain and range of the function.
- d. Determine whether each parabola has a maximum or minimum value, and determine this value.
- e. Identify the *x*-intercept(s) and *y*-intercept





1.2 X-intercepts, Zeros, and Roots

To find the *x*-intercepts of a quadratic function using a calculator, follow these steps:

- Graph the function in Y₁=.
- Press GRAPH.
- Make sure you can see both *x*-intercepts on the screen if not, change your window!
- Press 2nd Calc Zero.
 - Move your cursor to the left of the *x*-intercept you want to find and press enter.
 - Move your cursor to the right of the *x*-intercept you want to find and press enter.
 - Press enter again.

Example 1

Find the *x*-intercepts of the following quadratic functions. Sketch a graph of each function. LABEL each *x*-intercept.



You can also solve quadratic equations by graphing the corresponding quadratic function.

• The roots of a quadratic equation are the *x*-intercepts of the graph of the corresponding quadratic function. They are also the zeros of the corresponding quadratic function.

Example 2: Solving Quadratic Equations in Standard form

Solve each equation by graphing the corresponding function and determining the zeros.



Standard form of a quadratic equation:

What happens if a quadratic equation is not in standard form?

Example 3: Rearranging Quadratic Equations Rearrange the following quadratic equations into standard form.

a. $x(7-2x) = x^2 + 1$ b. 4x(x+3) = 3(4x+3)

Example 4: Solving a quadratic equation in non-standard form Determine the roots of this quadratic equation.

 $3x^2 - 6x + 5 = 2x(4 - x)$

For any quadratic equation, there can be zero, one, or two real roots. This is because a parabola can intersect the x-axis in zero, one, or two places.



Investigate:

Graph the following functions and determine the *x*-intercepts. What do you notice?



When a quadratic function is written in factored form each factor can be used to determine a zero of the function by setting each factor equal to zero and solving.

- If a parabola has one or two *x*-intercepts, the equation of the parabola can be written in factored form using the *x*-intercept(s) and the coordinates of one other point on the parabola.
- Quadratic functions without any zeros cannot be written in factored form.
- If a quadratic function has only one *x*-intercept, the factored form can be written as follows:

Example 1: Factoring Review

Greatest Common Factor:

 $-4x^2 + 12x$

Difference of Squares:

 $x^2 - 49$

Trinomial, a = 1

 $x^2 + 8x + 16$

Trinomial, $a \neq 1$

 $6x^2 + 13x + 6$

Example 2:

Determine the *x*-intercepts of the following functions. a. y = (x - 3)(x + 7)

b. $y = x^2 - 49$

c.
$$y = x^2 - 14 + 45$$

Example 3:

The *x*-intercepts of a function are 4 and -7. What is the equation of the function if the parabola is of standard width?

Example 4: Determine the equation of a quadratic function, given its graph

- a. Determine the function that defines this parabola.
 - i. Write the function in factored form.
 - ii. Write the function in standard form.



- b. Determine the function that defines this parabola.
 - i. Write the function in factored form.
 - ii. Write the function in standard form.



1.4 The Vertex

You can determine the coordinates of the vertex of a quadratic function by following these steps on your calculator:

- Graph the function in Y₁=.
- Press GRAPH.
- Change the window so you can see the vertex (if needed).
- Determine if the vertex is a maximum or a minimum.
- Press 2nd CALC and then either MAXIMUM or MINIMUM depending upon what your graph looks like.
- Move your cursor to the left of the vertex and press enter.
- Move your cursor to the right of the vertex and press enter.
- Press enter again. Your coordinates of the vertex will be displayed.

Example 1:

Determine the coordinates of the following quadratic functions using your calculator. State whether each vertex is a maximum or a minimum.

a.
$$y = 3x^2 - 2x + 5$$

b. $y = x^2 - 4$

c.
$$y = -2x^2 + 7x - 3$$

Example 2: Locating the Vertex Using Technology

A skier's coach determines the quadratic function that relates the skier's height above the ground, *y*, measured in metres, to the time, *x*, in seconds that the skier was in the air: $y = -4.9x^2 + 15x + 1$.

Graph the function. Then determine the skier's maximum height, to the nearest tenth of a metre, and state the range of the function for this context.



Axis of Symmetry Review!

The axis of symmetry is located in the middle of the parabola. What else is located in the middle of the parabola?

If you know the location of the axis of symmetry, you will be able to determine the location of the vertex.

Example 3:

A quadratic function $y = -3x^2 + 4x - 9$ has an axis of symmetry at x = 1.

a. Determine the coordinates of the vertex.

b. Determine if the vertex is a maximum or a minimum.

Given the location of the two *x*-intercepts, you are also able to determine both the location of the axis of symmetry. This is because parabolas are

Example 4

Quadratic functions have the following *x*-intercepts. Determine the location of each axis of symmetry.

a. x = 3 and x = -7

b. x = -2 and x = -8

c. x = 4 and x = 8

1.5 Sketching Graphs of Quadratic Functions

A properly labeled sketch of a quadratic function has the following characteristics:

- Labeled vertex
- Labeled *x*-intercept(s)
- Labeled *y*-intercept

Example 1: Graphing a Quadratic Function Using a Table of Values

Sketch the graph of the function $y = x^2 + 2x - 3$.

x	у
-3	
-2	
-1	
0	
1	



Determine the following:

- *y*-intercept
- *x*-intercepts
- equation of the axis of symmetry
- coordinates of the vertex
- domain
- range

Example 2: Graphing a quadratic function given in factored form

Sketch the graph of the quadratic function: f(x) = 2(x + 1)(x + 6)

Zeros:

y-intercept:

Equation of the axis of symmetry:

Vertex:

Domain:

Range:

Max/Min:

(1) direction of opening:		y							
(2) vertex?		10							
(3) max/min value?		-8-							
(4) <i>y</i> -intercept?		-6							
		-4							
(5) zeroes?		-2							
		╀	2	4	-	6	 3	x 1	
		-2							
(6) domain:		1							
(7) range:									
(8) axis of symmetry :									

Example 3: Analyze the following quadratic function: $y = \frac{1}{2}(x - 4)^2 - 2$



1.6 Solving Problems Using Quadratic Models

Example 1: You are an astronaut on the moon. You hit a golf ball with your golf club. The height of the ball, h(t), in metres, over t, in seconds, could be modeled by the function: $h(t) = -0.81t^2 + 5t$. What is the maximum height of the ball?

Example 2: The engineers who designed the Coal River Bridge on the Alaska Highway in BC used a supporting arch with twin metal arcs. The function that describes the arch is $h(x) = -0.005061x^2 + 0.499015x$ where h(x) is the height (in meters) of the arch above the ice at any distance, x (in meters) from one end of the bridge.

a. Determine the distance between the bases of the arch.

b. Determine the maximum height of the arch to the nearest tenth of a meter.

Example 3: The flight time for a long-distance water ski jumper depends on the initial velocity of the jump and the angle of the ramp. For one particular jump, the ramp has a vertical height of 5 m above water level. The height of the ski jumper in flight, h(t), in metres, over time, t, in seconds, can be modelled by the following function: $h(t) = 5.0 + 24.46t - 4.9t^2$

- a. How long does this water ski jumper hold his flight pose?
- b. When is the ski jumper 21 m above the ground?
- c. What height is the ski jumper 3s after he takes off?

Example 5: The height of a baseball can be modeled by the function $f(x) = -4(x-3)^2 + 10$, where *x* is the time in seconds and *f*(*x*) is the height in metres.

a) When does the baseball reach its maximum height?

b) What is the maximum height of the baseball?